A New and Easie

METHOD

TO THE

ART

OF

DYALLING.

CONTAINING

I. All Horizontal Dyals, all Upright Dyals, Reflecting Dyals, Dyals without Centres, Nocturnal Dyals, Upright Declining Dyals, without knowing the Declination of the Plane.

II. The most Natural and Easie Way of Describing the Curve-Lines of the Suns Declination on any Plane.

The like never before Published.

By *THOMAS STRODE*, Esq; of *Maperton* in the County of *Somerset.*

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2724:11

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ERRATA.

ERRATA.

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FINIS.

ADVERTISEMENT.

NAvigation, and all the Parts of the Mathematicks, are taught in *Engliſh* and *French* by Mr. *Reeve Williams*, at the *Virginia Coffee-Houſe* in St. *Michael's-Alley*, in *Cornhil*, near the *Royal Exchange*, *London*.

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Removed from the Poultry.

TO THE READER.

Courteous Reader,

THis was chiefly composed for some near Relations; but the Method being Natural, Easie, and not Common, I think I ought to communicate, and not to Bury it with me. I do not Publish this as to prefer it before Trigonometrical Calculation of Dyals, for no way can be Exacter than that; but this is to save that Labour in Young Beginners, lest they should be dejected with the tediousness of the other: And this Discourse but little short in Exactness with that. For though we can Calculate Dyals to Minutes, yea to Seconds, yet when we come to Delineate them, we can be no Exacter than the Line of Chords is, which seldom exceeds a Foot; but in Reflecting Dyals, and some others, we have sometimes Tangents for Radius's, of five times that length: Neither can Dyals, whose Centre is not in the Plane, be well drawn without two Tangents, unless they are first drawn on another Plane; and this is some help to Calculation, for here is Upright Declining Planes, the Sub-stiles Distance, the Stiles Heighth, and Inclination of the Meridians are already Calculated, and the Angle each Hour makes with the Sub-stile at the Pole must first be found before we can find the Angle on the Plane with the Sub-stile: The chief Difference is, that in the one, the Angle on the Plane is to be measured with a Line of Chords, in the other, the Angle at the Pole is to be measured with a Line of Tangents.

If you have on a Ruler a Line of Chords, Sines, Tangents, and Secants for a small Radius, as an Inch and an half, two,

as

or two and an half, you may use that instead of the Table of Sines, Chords, Tangents and Secants; but if it be a greater Radius, you will find an Inconveniency of describing Dyals on a small Plane.

The Method of drawing the Parallels of the Suns Declination, will give (if I am not deceived) a Diversion to those of the Highest Forms in the Mathematicks, being not Extant (as I have great reason to believe) in any Author; being grounded on such Paradoxes in the Conicks, That some of them on the first View will hardly be admitted; for with Gratefulness, I do acknowledge, That I received Seeds and Plants from some eminent Persons of the Royal Society, since deceased; but being removed into my barren Soil, and Cultivated and Manured according to the new Husbandry, produced Fruit beyond my Expectation, whereof this is part; besides their Geometrical way of Delineating their Curves, I have for the Latitude of London, *in a Direct South Dyal, given the Distances of the Points of the Hyperbole on Parallels to the Transverse Axe, from the Conjugate, which by a Diagonal Scale will quickly be placed.*

Signs used in this Treatise.

= stands for Equal
|| stands for Parallel
:: stands for Proportion
∷ stands for Continued Proportion
□ stands for Parallelogram
△ stands for Triangle

∠ stands for Angle
S. stands for Sine
C. S. stands for Co-sine
T. stands for Tangent
C. T. stands for Co-tangent.

A New and Easie METHOD TO THE Art of Dyalling.

CHAP. I.

Of Dyalling in General.

DYalling is an Art teaching how to Measure the time of the Day, by the Shade of the Sun.

The Heavens, with the Sun and Stars, go round the Earth every Twenty four Hours, with an Equal Motion. Two Points of the Heavens are Immovable, which are called the Poles; and there is an Imaginary Line passing from one Pole to the other, which is called the Axe of the World. The Circle which cuts the Axe at Right-angles, and divides the Heavens into two Equal Parts between the said Poles, is the Equinox or Equator, which (as every great Circle) is divided by Astronomers into 360 Equal Parts, called Degrees; of which the Sun goes 15 Degrees each Hour: Every Degree is divided into 60 Minutes, as by the Globe may be seen.

In every Dyal, that which gives the Shade, and shews the Hour, is called the Gnomon or Stile, and must always be parallel to the Axe of the World; which to place exactly, is the chiefest thing in Dyalling. The Line under it, on the Plan, is called the Substile.

The Angle each Hour makes with the Stile or Axe, which is commonly called the Angle at the Pole, is always equal to 15 *deg.*

to 1 *hour* = 2 *hours* = 30 *degrees* 3 *hours* = 45 *degrees*. Therefore on Plans which cut the Axe at Right-angles, which are uſually called Equinoctial Plans, it is but dividing a Circle into 24 equal Parts, and let a Pin ſtand perpendicular to the Plan on the Centre to repreſent the Axe, and to give the Shade; and it is a true Dyal.

But if the Axe be cut with a Plane at Oblique-angles, then the Stile ſtands leaning, and the Hours on the Plane are unequal. To find the true Meaſure of which, is the Subject of Dyalling.

Therefore the Plane of every Dyal muſt firſt be compared with the Horizon and Meridian.

The Horizon is that part of the Heavens as a quick Eye can diſcern in an open place, which divides the Heavens into two Equal Parts. The middle Point of the Heavens which is over our Heads, is called the Zenith; and that under our Feet the Nadir.

The Meridian is a great Circle of the Heavens paſſing through the Poles, the Zenith and Nadir, in which the Sun is at Noon, and at Twelve a Clock at Night.

I. If the Plane of the Dyal be parallel to the Horizon, it is an Horizontal Plane.

II. If the Plane makes Right-angles with the Horizon, it is an Erect or Upright Dyal.

III. If an Obtuſe-angle with the Horizon, it is an Inclining Plane; and what it wants of a Semi-circle, is the Angle of Inclination; or it is the Angle made by the Perpendicular and the Axe of the Horizon.

IV. If an Acute-angle, it is a Reclining Plan. I do follow Mr. *Gunter* and Mr. *Wingate*: Others, as Mr. *Oughtred*, Mr. *Wells*, Mr. *Newton*, and Mr. *Collins*, do call my Inclining a Reclining Plane, and my Reclining, an Inclining Plane.

Some call one the Lower Face, and the other the Upper Face. How to find the Angle of Inclination, I will ſhew in the laſt Chapter.

V. If an upright Plane does reſpect the South, ſo that the Meridian is perpendicular to the Plane, it is called an Erect, Direct South-Plane. In this the Meridian is the Subſtile.

VI. If the Meridian-Line and the Perpendicular to the Plane be divers, the Plane is ſaid to decline; and the Angle made by the Meridian and the Planes Perpendicular, is the Angle of the Declination of the Plane.

If

If the Meridian-line be on the { Right / Left } Hand of the Perpend. the Plan declines { Westward. / Eastward.

If an upright Plane has the Sun shining on it at Noon, it is a South Plane, and the Centre of the Dyal is above; if it has not, it is a North Plane, and the Centre of the Dyal is below.

VII. If a Plane be neither Erect to the Horizon, nor Direct to the South or North, it is called a Declining Inclining Plane, or a Declining Reclining Plan.

VIII. If a Plane be parallel to any great Circle that passes through the Poles, it is called a Polar Plan, and is parallel to some one of the Hours; so that the Pole is neither elevated above, nor depressed below the Plane, and consequently the Dyal has no Centre, and therefore its Stile, Substile and lower Lines, are Parallel.

If it be parallel to the Hour of Six, it is a Direct Polar Plane, and the Substile is the Meridian, and its Inclination is equal to the Elevation of the Pole.

If to the Meridian, it is an East or West Dyal, and its Substile is the Hour of Six.

1. Observe all Angles are usually measured by a Line of Chords, where Radius or Semi-diameter is equal to the Chord of 60°. and do contain 90°. which is a Quadrant; sometimes by a Tangent-line.

2. If the Hours of a Dyal be never so truly drawn, if the Stile be not well plac'd, it will not go exact; therefore observe these Three Rules in setting your Stile.

1. That it has the same Centre (if there be any) with the Substile and Hours.

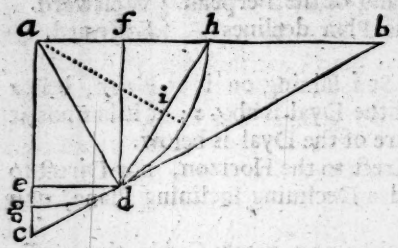
2. Its due Angle.

3. That it stands perpendicular to the Substile.

CHAP. II.

The Nature of Sines, Chords, Tangents and Secants, with a Table of them for each Degree.

IN the ensuing Page is a Table for the Sines, Chords, Tangents and Secants of each Degree of the Quadrant. Therefore I will first shew what Sines, Chords, Tangents and Secants are.



Let *a g d h* be the Quadrant of a Circle, *d a h* any Angle: Here it is equal to 60, whoſe meaſure is the Arch *d h*. Make *d f* parallel to *a g*, and *d e* parallel to *a d*; to *a d* apply the perpendicular *c d b*. Suppoſe *a d* the Radius to be Equal to 10000, then *d b* is the Tangent of the Angle *d a h* = 60 = 17320; *d f* is the Sine of the ſame Angle = 8660; *a b* is the Secant of the ſame = 20000; and *d h* is the Chord = to double the Sine of *d i* = 30; half the Arch = 2, in 5000 = 10000.

The Angle *g a d* is the Complement of the Angle *d a h*; *i. e* what it wants of 90° = 30°; and *d c* is its Tangent = 5773; *d e* is its Sine = 5000, and *a e* is its Secant = 11547, and *d g* is its Chord = to double the Sine of 15° half the Arch = 5176, and the ſame Lines are known if you compare them with the Arch *d h*, by the names of the Co-tangents, Co-ſines, Co-ſecants, and Complements of the Arch *d h* = 60°. Obſerve, in this Figure here are ſeven like Triangles, *viz. b a c, b d a, c d a, h f d, d f a, a e d*, and *d e c*; from whence ariſes ſeveral Proportions and continued Proportions by 8 *Euc.* 6.

1. *c d.* *d a.* *d b* ∴ ∴
Tangent. Radius. Co-tangent. are in continued Proportion.

2. *a f.* *a d.* *a b.* ∴ ∴ So *a e.* *a d.* *a c.*
Cō-ſine. Radius. Secant. Sine. Radius. Coſecant.
are in continued Proportion.

3. *a d.* *d b* :: *d e.* *a e* :: *a c.* *a b.*
Radius. Tangent. Co-ſine Sine. Co-ſecant. Secant.

4. *a d.* *d c* :: *a e.* *d e* :: *a b.* *a c.*
Radius. Co-tangent. Sine. Co-ſine. Secant. Co-ſecant.

2. In every plain rectangle Triangle, if one Leg be the Radius, the other Leg is the Tangent of its oppoſite Angle; and the Hypothenuſe is the Secant of the ſame Angle.

3. If the Hypothenuſe be the *Radius*, each Leg is the Sine of its oppoſite Angle.

If

RADIUS 1000.

0	Sines	Cord.	Tang.	Secant.	90	Co-si.	Com. C	Co-tang.	Cosecant
1	174	174	174	10001	89	9998	14018	572900	572987
2	349	349	349	10006	88	9994	13893	286362	286537
3	523	523	524	10014	87	9986	13767	190811	191073
4	697	698	699	10024	86	9975	13640	143007	143356
5	871	872	875	10038	85	9962	13512	114300	114737
6	1045	1046	1054	10055	84	9945	13382	95144	95668
7	1219	1221	1228	10075	83	9925	13252	81443	82055
8	1392	1394	1405	10098	82	9903	13120	71154	71853
9	1564	1569	1584	10124	81	9877	12889	63137	63924
10	1736	1743	1763	10154	80	9848	12989	56713	57588
11	1908	1917	1944	10187	79	9816	12721	51445	52408
12	2079	2090	2125	10223	78	9781	12586	47046	48097
13	2249	2264	2309	10263	77	9744	12450	43315	44454
14	2419	2488	2493	10306	76	9703	12313	40108	41735
15	2588	2610	2679	10353	75	9659	12175	37320	38637
16	2756	2784	2867	10403	74	9612	12036	34874	36279
17	2924	2956	3057	10457	73	9563	11896	32708	34203
18	3090	3128	3249	10514	72	9510	11756	30777	32360
19	3255	3301	3443	10576	71	9455	11614	29042	30714
20	3420	3473	3640	10642	70	9397	11472	27475	29238
21	3583	3644	3838	10711	69	9336	11328	26051	27904
22	3746	3816	4040	10785	68	9272	11184	24751	26695
23	3907	3887	4245	10863	67	9205	11039	23558	25593
23.3	3987	4073	4348	10904	66	9170	10966	22998	25078
24	4067	4158	4452	10946	66⅓	9135	10892	22460	24586
25	4226	4328	4663	11034	65	9063	10745	21445	23662
26	4384	4499	4877	11126	64	8988	10598	20503	22812
27	4540	4669	5094	11223	63	8910	10449	19626	22027
28	4695	4838	5317	11326	62	8829	10300	18807	21300
29	4848	5007	5543	11433	61	8746	10150	18040	20626
30	5000	5176	5773	11547	60	8660	10000	17320	20000
31	5150	5345	6008	11666	59	8571	9848	16643	19416
32	5299	5512	6248	11791	58	8480	9696	16003	18871
33	5446	5680	6194	11923	57	8386	9543	15398	18361
34	5572	5818	6745	12062	56	8290	9390	14825	17883
35	5736	6004	7002	12207	55	8191	9235	14281	17434
36	5878	6180	7265	12361	54	8090	9080	13764	17013
37	6018	6346	7534	12521	53	7986	8923	13270	16616
38	6156	6510	7813	12690	52	7880	8768	12799	16242
39	6293	6676	8098	12867	51	7771	8610	12349	15890
40	6428	6840	8391	13054	50	7660	8452	11917	15557
41	6560	7004	8693	13250	49	7547	8293	11503	15242
42	6691	7166	9004	13456	48	7431	8134	11106	14945
43	6820	7330	9325	13673	47	7313	7975	10723	14663
44	6946	7494	9651	13901	46	7193	7814	10355	14295
45	7071	7654	10000	14142	45	7071	7654	10000	14142
	Co-si.	Co.C.	Co-tan	Co-sec.		Sines	Cords	Tangent.	Secants.

If you have a Diagonal Scale for an Inch and half an Inch, this Table will ſerve for Radius of 10 Inches, 5 Inches, one Inch, and half an Inch; and if you double the Numbers it will ſerve for a Radius of 20, or 2 Inches; if you will half the Numbers, it will ſerve for a Radius of 2 half Inches, or one quarter of an Inch.

If you deſire to find the Sine, Chord, &c. of any degree, for any other Radius, ſuppoſe 3 Inches, then you muſt work by the Rule of Three; as what is the Sine, Chord, Tangent and Secant for 20 Degrees, the Radius being 3 Inches = 300, Say,

		Sine	Sine	Cord	Cord	Tang.	Tang.
10000.	300 ::	3420.	102,6 ::	3472.	104,1 ::	3640.	109,2.

Secant	Secant.
10642.	319,2.

If you deſire to find the Sine, Chord, &c. for any Minute, as what is the Secant of 38° 50′, take the difference of its neareſt Secants, *viz.* 39, 38, which is 00177, then ſay, 60, 50 :: 00177, 00147, which added to the Secant of 38 = 12690, the ſumm 12837 is the Secant of 38, 50, which was deſired. Its Co-ſecant is 15947, its Chord is 6648, or by the Tables of Natural Sines, Tangents and Secants, or the Table of Artificial Sines and Tangents by finding the Natural Numbers of each Artificial Sine and Tangent, as is directed in moſt Authors.

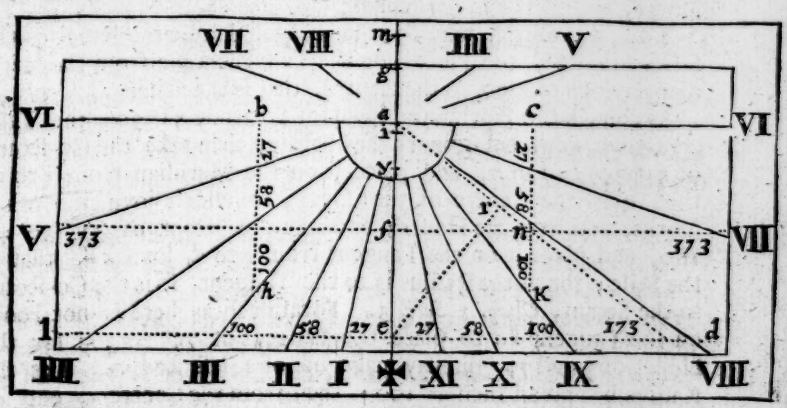
CHAP. III.

How from this Table, and a Diagonal Scale, or Scale of Equal Parts, to make an Horizontal Dyal for any Latitude.

ON the Line *l, e, d*, erect the Perpendicular *e a*, which is the Meridian, and the Line *l e d* is a Tangent Line, on which place the Tangent of every 15 Degrees from the Meridian, as they are in the Table in the Margent, Page 5. As the Radius being equal to an Inch = 100, place the Tangent of 15° = 26,8

on

on the Line *l e d* from the Meridian on each Side of it, as *e* 1, *e* 11, for the Hours of 1 and 11, and the Tangent of 30° = 57,7, on the ſame Line, on each Side of the Meridian, as *e* 2 *e* 10, for the Hours of 2 and 10, and the Tangent of 45° = 100, on the ſame Line on each ſide of the Meridian, as *e* 3, *e* 9, for the Hours of 3 and , and ſo the Tangent of 60 = 173 for 4 and 8, and the Tangent of 75 = 373, for 5 and 7.



Rad. 1000
Sine 782
Co-ſine 622
51° 30′ Sec. 1606
Co-ſec. 1277

hor.	Ang. Pole	Tang	hor.	Ang. Pole	Tang.
12	° ′		3 0	45 .	1000
1	3 . 45	65	1	48 . 45	1140
2	7 . 30	131	2	52 . 30	1303
3	11 . 15	199	3	56 . 15	1496
1 0	15 . 0	268	4 0	60 .	1732
1	18 . 45	339	1	63 . 45	2028
2	22 . 30	414	2	67 . 30	2414
3	26 . 15	493	3	71 . 15	2946
2 0	30 .	577	5 0	75 .	3732
1	33 . 45	668	1	78 . 45	5027
2	37 . 30	767	2	[illegible] . 30	7595
3	41 . 15	877	3	[illegible] . 15	15257

From

From this Tangent, ſo marked, may be made any Horizontal or Direct South or North, whether Upright, Inclining or Reclining for any Latitude, by the true placing the Centre of the Dial, which is always found by taking the Co-ſecant of the Stiles Altitude above the Plane in equal Parts by the ſame Diagonal, and for the ſame Radius, and placing it on the Meridian from the Tangent-Line for the Centre, then connect the Centre to thoſe Marks, and you have the Hours.

To find the Stile, take on your Diagonal Scale the Secant of the Stiles Altitude, and place it on the Tangent-Line from the Meridian, which connect to the Centre, that is the Stile.

As to make an Horizontal Dyal for Latitude 51° 30′, the Tangent-Line *l e d*, being mark'd as before, then take the Co-ſecant of 51° 30′ = 127.7, and place it on the Meridian from *e* to *a*, then *a* is the Centre of the Dyal, to which joyn the other Marks, you have the Hours; and take the Secant of 51° 30′ = 160, and place it on the Tangent from *e* to *d*, joyn *a d*, that is the Stile; for as the Radius is to the Tangent, ſo is the Co-ſecant to the Secant, *Chap.* 2. *Sect.* 3. Foraſmuch as here is not room on the Tangent *l d* to place the Tangent of 75° = 373 for the Hours of 5 and 7, you may make uſe of a leſs Radius. Let the Radius be half an Inch = 100; then from the Centre *a*, put *a f* on the Meridian = 127,7, the Co-ſecant of the Stiles Altitude 51° 30′, and at *f* apply another Tangent-Line; then on the Diagonal Scale, on the half Inches, take the Tangent of 75 = 373, and place it on that Tangent-Line on each Side of the Meridian, as *f* 5, *f* 7, joyn *a* 5, *a* 7, thoſe are the Hours for 5 and 7; If you continue them through the Centre, they will ſerve for 5 in the Morning and 7 in the Evening; ſo you muſt continue the Hours of 4 and 8, and through the Centre draw a Perpendicular to the Meridian for the Hours of 6.

An Erect Direct South Dyal, Lat. 51°. 30′.

If you will make a Direct South-Dyal, the Tangent being made as before, for that the Stiles heighth is 38° 30′ above the Plane, you muſt take its Co-ſecant = 160.6 and place it on the Meridian from *e* to *g*, where is the Centre of your Dyal, which connect to thoſe marks on the Tangent-line, and you have the Hours from 8 to 4 For the Hours of 7 and 5 (the Tangent being not long enough) make

make use of a less Radius, *viz.* half an Inch. Then take of that Radius the Co-secant of 38° 30′ = 160, and apply another Tangent; whereupon measure 373 the Tangent of 75° for the Hours of 5 and 7. Then for the Stile, put the Secant of the Stiles Altitude 38° 30′ = 128 on the Tangent-line from the Meridian, which being joyned to the Centre, is the Stile.

Of Placing the Hours.

In every Dyal truly placed, if you stand on the South-side of the Plane looking Northward, the Hours on your Left-hand of the Meridian are the Morning Hours; on your Right-hand are the Evening Hours; but if you stand on the North of the Plane, your Face being Southward, then the Forenoon Hours are on your Right-hand, and the Afternoon Hours on your Left-hand; for your Right-hand in relation to the Plane, is where your Left-hand was.

An Horizontal Dyal for Lat. 56°.

III. If you are to make an Horizontal Dial for *Edenburgh*, where the Latitude is near 56°, then the Tangent-line being divided as before, take the Co-secant of the Stiles heighth 56° = 120,6, and place it on the Meridian from *e* to *i*: Then at *i* is the Centre of the Dyal, which connect to the Marks on the Tangent-line, and you have the Hours. For the Stile you must take the Secant of 56° = 178,8, and set it on the Tangent-line, which if you joyn to the Centre *i*, you have the Stile.

An Erect, Direct South Dyal, for Latitude 56 *Deg.*

IV. The Tangent being divided as before, take the Co-secant of the Stiles Altitude 34 *d.* = 178,8, and put it from *e* to *m*, then *m* will be the Centre of that Dyal, to which joyn the Marks on the Tangent, and you have the Hours. For the Stile put on the Tangent, the Secant of 34 *d.* = 120, which connected to *m* is the Stile.

If the Centre of the Dyal be given; then take the Co-secant of the Stiles Altitude in equal Parts, and place it from the Centre on

the Meridian, where apply a Perpendicular for a Tangent, and mark it as before.

Of Direct South Dyals, Inclining and Reclining to the Horizon.

V. After the ſame manner, all Direct South-Dyals are made, whether the Planes be inclining or reclining, obſerving this Rule in placing the Stile.

If it be a direct South-Plane inclining to the Horizon, *vide Cap.* 1. *Sect.* 3. then the difference of the Angle of Inclination, and the elevation of the Pole is the Heighth of the Stile above the Plane.

If the Poles elevation be	greater / leſs	than the Angle of Inclination, then the	North / South	Pole is elevated, & the Centre is	below / above

If Equal, then it is a Polar Plane.

If it be a Direct North-Plane inclining to the Horizon, then the Summ of the Angles of Inclination and Elevation of the Pole is the Heighth of the Stile above the Plane. If the Angles be equal to 90 *d.* then it is an Equinoctial Plane, being parallel to the Equinox, whoſe Gnomon ſtands Perpendicular to the Plane, and its Hours are equally divided, each Hour being 15 Degrees.

Obſerv. The Reclining Dyals are to be made in the ſame manner, and that a South Inclining Dyal is a North Reclining Dyal; and a South Reclining Dyal is a North Inclining Dyal: In all theſe, the Meridian and Subſtile are the ſame.

Demonſtration.

Fig. I. At the interſection of the Tangent with the Sub-ſtile, erect a Perpendicular to the Stile, as *e r*, to which make *e y* on the Subſtile Equal; then *e y* is the Radius on which a Circle is to be deſcribed, which repreſents the Equinox, and is to divide the Tangent-line into Hours: To avoid which Labour, *e y* being equal to the Radius, we do take the Tangents from the Table.

Then if the Line *a e* be the Radius, the Line *e r*, or *e y* is the Sine of the ∠ *e a r*, the Stiles heighth, and *e* 1 or *e* 11, is the Tangent of the Angle on the Plane.

If

If the Line *e r* or *e y* be the Radius, then the Line *a e* is the Secant of the $\angle a e r$, and Co-secant of the Angle *e a r* the Stiles heighth, and *e d* is the Secant of the $\angle r e d$, or $\angle r a e$, the Stiles heighth, and *e* 1, *e* 11, is the Tangent of the Angle at the Pole with the Substile = Tangent 15 = 27: From whence follows,

As *e y* the Radius, is to *e y* the Sine; so is *e i* the Tangent of the Angle at the Pole, to *e i* the Tangent of the Angle on the Plane; which is the Rule in *Trigonometry* to find the Angles made by each Hour, and the Sub-stile on the Plane.

To make *e r* = to the Radius, we must make *a e* = to the Co-secant of the Stiles altitude. Let us try this Rule by Calculation.

	As the Radius is	10.000000
	to the Sine of the Stiles heighth 51° 30′	9.893544
For 1.	So is the Tangent of the Angle at the Pole 15°	9.428052
	to the Tangent of the Angle on the Plane 11° 51′	9.321596
For 2.	So the Tangent 30	9.761439
	to the Tangent 24 19	9.654983
For 3.	So the Tangent 45	10.000000
	to the Tangent 38 03.	9.893544
For 4.	So the Tangent 60	10.238561
	to the Tangent 53 35	10.132105
For 5.	So the Tangent 75	10.571947
	to the Tangent 71 06	10.465491

Here is one Inconvenience of Drawing the Hours by a Tangent on a small Plane, that the Half-hours next Six a Clock cannot be drawn; and on declining Planes, One Hour sometimes cannot: To avoid which, we may use Mr. *Cassel*'s Invention, of *Hart-Hall*.

Make a Rectangle Parallelogram, two of whose Sides are parallel to the Substile, and each Sides Distance from it, is equal to the Sine of the Stiles heighth above the Plane; then cross the Sub-stile at Right-Angles with two other Lines, one passing through the Centre, whose Distance must be equal to the Co-secant of the Stiles Heighth.

Then on the oppoſite Side to the Centre place the Tangent that each Hour makes with the Sub-ſtile at the Pole, as is before directed. Then for the other Hours, Take the Co-tangent of the Angle that each Hour makes with the Subſtile, and place it on the Sides parallel to the Sub-ſtile, from the Side paſſing through the Centre, which in the Horizontal, and Direct South-Dials is the Hour of Six; then joyn theſe Marks to the Centre, and your Hours are drawn.

For that the Side of the Parallelogram oppoſite to the Centre is not long enough to draw the Stile according to former Directions, take the Co-ſine of the Stiles heighth, and place it on one of the parallel Sides, which connect to the Centre, that is the Stile.

Fig. I. As on the Horizontal Dial, let *b c*, *k h* be the □, the Sides *b h*, *c k*, the Diſtance from the Sub-ſtile *a e* = to the Sine of the Stiles heighth 51° 30′ = 78, the Radius being 100 make *b h*, *c k*, = Coſecant of 51° 30′, the Stiles heighth = 128, then make *e* 11, *e* 1 = Tangent of 15° = 27, for 1 and 11 a Clock, and *e* 2, *e* 10 = Tangent of 30° = 58′ for two and 10 a Clock; and foraſmuch as on this Side *h k* no more can be placed than on the Sides of the □ *b h*, *c k*, make *b* 5 *c* 7 = Co-tangent of 75 = Tangent 15 = 27 Radius being 100, for 5 and 7, and *b* 4, *c* 8 = Co-tangent of 60 = Tangent, 30 = 58 for 4 and 8, and *b* 3, *c* 9 = Co-tangent of 45 = 100 for 3 and 9 a Clock; connect theſe to the Centre and you have the Hours: To draw the Stile, make *c n* = Co-ſine 51° 30′ = 62 joyn *a n*, that is the Stile.

Obſerv. Make uſe of as large a Radius, to meaſure theſe Lines, as your Plane will bear; and by the ſame Radius you meaſure your Co-ſecant, you muſt meaſure your Tangents, and by a longer Radius you may meaſure your Sines and Co-tangents, for in all upright Dials your Plane will bear it.

Demonſtration.

Fig. I. In the Horizontal Dial in the △ *a c n*, you will find that *a c* being the Radius, *c n* is the Tangent of the ∠ *c a n*, and Co-tangent of the Angle *e a n* the Stiles heighth, and *a c* is made equal to the Sine of the ∠ *e a n* the Stiles heighth. Therefore as the Radius *a c* is to the Tangent *c n*, ſo is the Sine *a c* to Co-ſine *c n* by *Chap.* 2. *Sect.* 4. ſo the Angle *c a n* is equal to the Complement of the Elevation of the Pole.

In

In the △ *a c* 7, *a c* being the Radius *c* 7, is the Tangent of the ∠ *c a* 7, equal Tangent 15 = 27, and Co-tangent ∠ 7 *a e* the Angle on the Plane with the Sub-ſtile, and *a c* be the Sine, then *c* 7 is the Co-tangent of the Angle at the Pole with the Sub-ſtile. Therefore,

As the Sine *a c* is to the Radius *a c*, ſo is the Co-tangent of the Angle at the Pole with the Sub-ſtile to *c* 7 the Co-tangent of the Angle on the Plane with the Sub-ſtile; as by Calculation you will find.

	As the Sine of 51° 30′	9.893544
	is to the Radius	10.000000
For 1	So is the Co-tangent of 15°	10.571947
	to the Co-tangent 11° 51′	10.676403
For 2	So is the Co-tangent 30°	10.238561
	to the Co-tangent 24° 19′	10.345017
For 3	So is the Co-tangent 45°	10.000000
	to the Co-tangent 38° 03′	10.106456
For 4	So is the Co-tangent 60°	9.761439
	to the Co-tangent 53° 35′	9.867895
For 5	So is the Co-tangent 75°	9.428052
	to the Tangent 71° 06′.	9.534507

How to make a Dyal on an Erect Declining Plane.

VI. What an Erect Declining Plane is, *vid. Chap.* 1. *Sect.* 6. In Declining Planes, before the Hours can be drawn, there are Three things requiſite to be found; that is, 1*ſt.* The Subſtiles Diſtance from the Meridian. 2*ly.* The Stiles Heighth above the Subſtile. 3*ly.* The Angle at the Pole made by the Subſtile and the Meridian, which is called in theſe Tables Enſuing, the Inclination of the Meridian, by ſome the Difference of the Meridians; all which are exactly found by Calculation, and therefore in the following Pages I have preſented you with two Tables calculated, the one for the Latitude of 51° 15′; the other for *London*, whoſe Latitude is 51° 30′; which Tables with taking their Difference, and working by the Rule of Proportion, will ſerve for any Latitude in *England*, without any ſenſible difference.

As

As what is the Subſtiles Diſtance, the Stiles Heighth, and the Inclination of the Meridians at *Cambridge*, where the Pole is elevated 52° 20′ for 30° of Declination.

		Subſtiles Diſt.	Stiles Heighth	Inclinat. Merid.
	51° 15′			
Firſt Table Lat.	15° . 51′	21° . 52′	32 . 49	36 . 31
Second Tab. Lat.	51 . 30	21 . 41	32 . 37	36 . 25
Their Difference	15	11	12	6

Cambridge Lat. 52° . 20′
London Lat. 51 . 30

Difference —— 00 . 50

Then ſay, if the Difference of the Latitudes of the Tables 15′ do give 50′, What ſhall 11′ give?

Subſtiles Diſtance	Stiles Heighth	Inclinat. Merid.
15. 50 :: 11.37 ::	12.40 ::	6.20

Which ſubſtracted from the Requiſites for the Latitude of *London*, you have the Requiſites for *Cambridge*.

	Subſt. Diſt.	Stiles Heighth	Inclin. Merid.
At *London*	21 . 41	32 . 37	36 . 25
Prop. Diſt.	37	40	20
At *Cambridge*	21 . 4	31 . 57	36 . 5

By Calculation thus.

Radius		10.00000
Co-ſine Declin.	30 00	9.93753
Co-ſine Elev.	52 20	9.78609
Sine Stiles Heighth	31 57	9.72360
Radius		10.00000
Sine of Declin.	30 00	9.69897
Co-tang. Elev.	52 20	9.88759
Tang. Subſt. Diſt.	21 06	9.58656
Radius		10.00000
Co-tang. Declin.	30 00	10.23856
Sine Elev.	52 20	9.89849
Co-tang. Inclin. Merid.	36 03	10.13705

Latitude 51 *Deg.* 15 *Min.*

decl.	*Subf. diſt.*	*Stil. beig.*	*Incl. Mer.*
1	0 . 48	38 . 45	1 . 17
2	1 . 36	38 . 44	2 . 34
3	2 . 24	38 . 41	3 . 51
4	3 . 13	38 . 38	5 . 08
5	4 . 1	38 . 34	6 . 24
6	4 . 50	38 . 30	7 . 41
7	5 . 36	38 . 25	8 . 57
8	6 . 23	38 . 19	10 . 13
9	7 . 10	38 . 11	11 . 28
10	7 . 56	38 . 3	12 . 44
11	8 . 42	37 . 54	14 . 0
12	9 . 29	37 . 45	15 . 15
13	10 . 14	37 . 35	16 . 29
14	10 . 59	37 . 24	17 . 43
15	11 . 44	37 . 13	18 . 58
16	12 . 29	36 . 59	20 . 11
17	13 . 13	36 . 46	21 . 24
18	13 . 56	36 . 32	22 . 37
19	14 . 39	36 . 17	23 . 49
20	15 . 22	36 . 02	25 . 01
21	16 . 03	35 . 45	26 . 12
22	16 . 44	35 . 28	27 . 23
23	17 . 25	35 . 10	28 . 33
24	18 . 05	34 . 52	29 . 43
25	18 . 45	34 . 33	30 . 53
26	19 . 24	34 . 14	32 . 01
27	20 . 02	33 . 54	33 . 09
28	20 . 40	33 . 33	34 . 16
29	21 . 17	33 . 12	35 . 24
30	21 . 52	32 . 49	36 . 31
31	22 . 28	32 . 27	37 . 37
32	23 . 03	32 . 03	38 . 42
33	23 . 37	31 . 39	39 . 47
34	24 . 11	31 . 15	40 . 51
35	24 . 44	30 . 50	41 . 55
36	25 . 15	30 . 25	42 . 58
37	25 . 47	29 . 59	44 . 01
38	26 . 18	29 . 33	45 . 03
39	26 . 48	29 . 06	46 . 04
40	27 . 17	28 . 39	47 . 05
41	27 . 46	28 . 11	43 . 06
42	28 . 14	27 . 43	49 . 06
43	28 . 42	27 . 14	50 . 06
44	29 . 09	26 . 46	51 . 05
45	29 . 35	26 . 16	52 . 03

dec	*Subf. diſt.*	*Stil. beig.*	*Incl. Mer.*
46	30 . 06	25 . 46	53 . 01
47	30 . 25	25 . 16	53 . 59
48	30 . 49	24 . 46	54 . 56
49	31 . 13	24 . 15	55 . 52
50	31 . 35	23 . 53	56 . 48
51	31 . 57	23 . 12	57 . 44
52	32 . 19	22 . 40	58 . 38
53	32 . 42	22 . 08	59 . 33
54	33 . 00	21 . 35	60 . 28
55	33 . 19	21 . 02	61 . 22
56	33 . 38	20 . 29	62 . 15
57	33 . 56	19 . 56	63 . 09
58	34 . 14	19 . 22	64 . 01
59	34 . 32	18 . 48	64 . 54
60	34 . 48	18 . 14	65 . 46
61	35 . 04	17 . 40	66 . 37
62	35 . 19	17 . 05	67 . 28
63	35 . 34	16 . 30	68 . 19
64	35 . 48	15 . 56	69 . 10
65	36 . 02	15 . 20	70 . 01
66	36 . 15	14 . 45	70 . 51
67	36 . 28	14 . 09	71 . 41
68	36 . 39	13 . 34	72 . 31
69	36 . 50	12 . 58	73 . 20
70	37 . 01	12 . 22	74 . 09
71	37 . 12	11 . 45	74 . 58
72	37 . 21	11 . 09	75 . 47
73	37 . 30	10 . 37	76 . 35
74	37 . 39	9 . 56	77 . 24
75	37 . 47	9 . 19	78 . 12
76	37 . 54	8 . 42	79 . 00
77	38 . 01	8 . 06	79 . 48
78	38 . 08	7 . 29	80 . 35
79	38 . 14	6 . 52	81 . 23
80	38 . 19	6 . 14	82 . 16
81	38 . 24	5 . 36	82 . 57
82	38 . 28	4 . 59	83 . 45
83	38 . 32	4 . 22	84 . 31
84	38 . 36	3 . 45	85 . 18
85	38 . 39	3 . 08	86 . 05
86	38 . 41	2 . 30	86 . 53
87	38 . 43	1 . 53	87 . 39
88	38 . 44	1 . 15	88 . 24
89	38 . 45	0 . 38	89 . 13
90	38 . 45	0 . 00	90 . 00

Lat. 51. 15.

Latitude 51 *Deg.* 30 *Min.*

dec.	Subs. dist.	stil. belg.	Incl. Mer.
1	0 . 48	38 . 29	1 . 17
2	1 . 36	38 . 28	2 . 23
3	2 . 23	38 . 27	3 . 49
4	3 . 11	38 . 24	5 . 07
5	3 . 58	38 . 20	6 . 23
6	4 . 45	38 . 15	7 . 39
7	5 . 32	38 . 10	8 . 55
8	6 . 15	38 . 04	10 . 11
9	7 . 05	37 . 57	11 . 27
10	7 . 52	37 . 49	12 . 43
11	8 . 37	37 . 40	13 . 57
12	9 . 23	37 . 30	15 . 10
13	10 . 08	37 . 20	16 . 26
14	11 . 06	37 . 09	17 . 40
15	11 . 38	36 . 58	18 . 54
16	12 . 22	36 . 43	20 . 07
17	13 . 04	36 . 32	21 . 20
18	13 . 49	36 . 18	22 . 33
19	14 . 31	36 . 03	23 . 45
20	15 . 13	35 . 48	24 . 57
21	15 . 55	35 . 33	26 . 09
22	16 . 35	35 . 16	27 . 19
23	17 . 16	34 . 57	28 . 29
24	17 . 56	34 . 39	29 . 38
25	18 . 34	34 . 21	30 . 47
26	19 . 13	34 . 01	31 . 56
27	19 . 50	33 . 42	33 . 04
28	20 . 29	33 . 20	34 . 12
29	21 . 05	32 . 59	35 . 19
30	21 . 41	32 . 37	36 . 25
31	22 . 15	32 . 15	37 . 31
32	22 . 50	31 . 52	38 . 36
33	23 . 25	31 . 27	39 . 45
34	23 . 59	31 . 04	40 . 40
35	24 . 31	30 . 40	41 . 49
36	25 . 04	30 . 14	42 . 52
37	25 . 35	29 . 48	43 . 55
38	26 . 04	29 . 22	44 . 58
39	26 . 35	28 . 56	45 . 59
40	27 . 03	28 . 29	47 . 00
41	27 . 33	28 . 01	48 . 00
42	28 . 01	27 . 33	49 . 00
43	28 . 29	27 . 05	50 . 00
44	28 . 55	26 . 36	50 . 59
45	29 . 14	26 . 07	51 . 57

dec	Subs. dist.	Stil. belg.	Incl. Mer.
46	29 . 46	25 . 37	52 . 55
47	30 . 11	25 . 07	53 . 53
48	30 . 35	24 . 38	54 . 50
49	30 . 58	24 . 06	55 . 46
50	31 . 21	23 . 35	56 . 42
51	31 . 45	23 . 04	57 . 38
52	32 05	22 . 32	58 . 33
53	32 . 26	22 . 00	59 . 28
54	32 . 46	21 . 28	60 . 23
55	33 . 06	20 . 55	61 . 17
56	33 . 24	20 . 22	62 . 10
57	33 . 42	19 . 49	63 . 04
58	34 . 00	19 . 06	63 . 57
59	34 . 15	18 . 42	64 . 49
60	34 . 33	17 . 56	65 . 41
61	34 . 47	17 . 34	66 . 33
62	35 . 05	17 . 00	67 . 24
63	35 . 18	16 . 25	68 . 16
64	35 . 34	15 . 50	69 . 07
65	35 . 43	15 . 15	69 . 57
66	36 . 00	14 . 40	70 . 47
67	36 . 13	14 . 05	71 . 38
68	36 . 25	13 . 29	72 . 27
69	36 . 36	12 . 53	73 . 16
70	36 . 46	12 . 18	74 . 06
71	36 . 55	11 . 41	74 . 55
72	37 . 06	11 . 06	75 . 44
73	37 . 15	10 . 29	76 . 33
74	37 . 24	9 . 53	77 . 21
75	37 . 32	9 . 16	78 . 09
76	37 . 40	8 . 40	78 . 57
77	37 . 47	8 . 03	79 . 46
78	37 . 53	7 . 27	80 . 33
79	37 . 59	6 . 49	81 . 21
80	38 . 04	6 . 12	82 . 09
81	38 . 09	5 . 35	82 . 56
82	38 . 14	4 . 58	83 . 43
83	38 . 17	4 . 20	84 . 31
84	38 . 21	3 . 44	85 . 18
85	38 . 23	3 . 06	86 . 05
86	38 . 26	2 . 28	86 . 52
87	38 . 28	1 . 52	87 . 39
88	38 . 29	1 . 15	88 . 26
89	38 . 29½	0 . 37	89 . 13
90	38 . 30	0 . 00	90 . 00

London Lat. 51. 30.

See the Explination in the following Page.

As Rad : S. Declin :: C.T. Elev. Subſtiles Diſtance.
As Rad : C. S. Declin :: C. S. Elev. Stiles Heighth.
As Rad : C.T. Declin :: S. Elev. C.T. Inclin. Meridians.

Directions for making an Erect Declining Dyal.

I. With a Line and Plummet, let fall a Perpendicular to the Horizon, which in all upright Dyals is the Hour of 12, if the Centre be on the Plane.

II. Then by the foregoing Tables, the Sub-ſtiles Diſtance being found, place it with a Line of Chords to the

Right } Hand of the Meridian } Weſtward
left } if the Plane declines } Eaſtward

For the Sub-ſtile is to ſtand } Weſt } among } Evening } Hours.
if the Plane Decline } Eaſt } the } Morning }

III. From the Centre take the Co-ſecant of the Stiles heighth, as in the Tables, and place it on the Sub-ſtile, where apply a Perpendicular for a Tangent, and another through the Centre.

IV. Finiſh your □ by drawing two Parallel Lines to the Subſtile, whoſe Diſtance is Equal to the Sine of the Stiles Altitude: Then take the Secant of the Stiles Altitude, and place it on the Tangent from the Sub-ſtile, which joyn'd to the Centre is the Stile; or its Co-ſine, and place it on the Parallel Side.

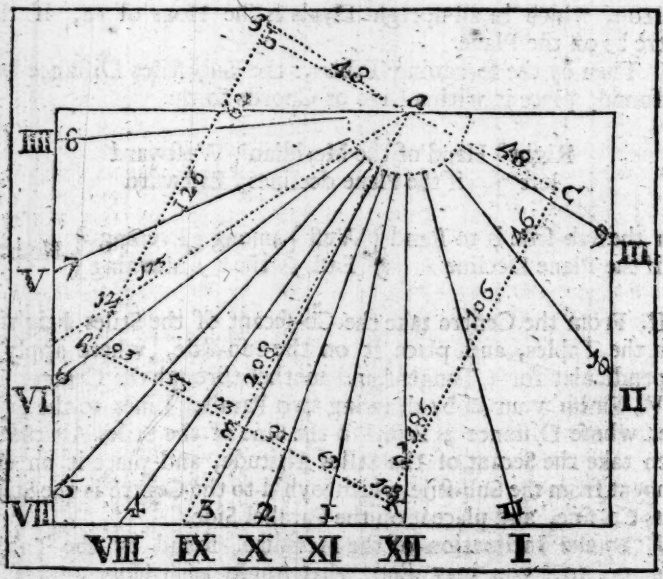
V. By the Inclination of the Meridian, found by the Table, page 15, 16, you may find what Angle each Hour makes at the Pole, with the Sub-ſtile, by ſubſtracting 15° from each Hour that is between the Subſtile and Meridian, from the Inclination of the Meridians, as you find it in the Table, and by adding 15° for the other Hours.

VI. Then take the Tangents of thoſe Angles by the firſt Table, and place them on the Tangent Line from the Sub-ſtile, then joyn thoſe Marks to the Centre, and your Hours are drawn. And for thoſe Hours that will not come on the Tangent, place their Co-tangent on the parallel Sides: Obſerve, every 6 Hours has the ſame Diſtance on the Parallel Side, as they have on the Tangent Line.

A South Erect Dyal, Declining Eastward 40°, *Lat.* 51° 15'.

Let *a d* be the Meridian, and Perpendicular to the Horizon, *a* the Centre of the Dyal; by the Table, page 15, you will find the Sub-stiles Distance = 27° 17', the Stiles Heighth, = 28 39 the Inclination of the Meridian = 47 05.

Fig. 3.



A North Erect Declining Eastward 40 degs.

Make the Angle *d a e* = Substiles Distance = 27° 17', either by a Line of Chords, or by the Table, the Chord being = 47 to the Left-hand, because the Plane Declines Eastward; then connect *a e*, that is, the Substiles, place the Co-secant of the Stiles Heighth 28° 39' = 208 on the Substile, the Radius being 100 from *a* to *e*, where apply a Perpendicular for a Tangent, as *h e k*, and finish your □ by drawing *b a c* through the Centre Parallel to *h e k*, and make *a b*, *a c*, *e h*, *e k* = 96, the Sine of 28° 39', the Radius being 200, joyn *b h*, *c k*, and your □ is finished.

Then

Hor.	∠ at Pole	Tang.
2	77 . 05	4360
1	62 . 05	1887
12	47 . 05	1075
11	32 . 05	627
10	17 . 05	307
9	2 . 05	36
Substile		
8	12 . 55	292
7	27 . 55	530
6	42 . 55	930
5	57 . 55	1595
4	72 . 55	3254
3	87 . 55	27489

	Radius	1000
	Sine	480
Stiles Heighth	Co-sine	877
	Secant	1140
28° 39′	Co-secant	2085

Then take the Distance of each Hour, as they are in the Margent, and place them on the Tangent Line from the Sub-stile, as make $e\,9 = 0:36 =$ Tangent 32 05, the Radius $= 100$, $e\,10 =$ Tangent 17 05 $= 30{,}7$, $e\,11 =$ Tangent 32 05 $= 62{,}7$, $e\,12 =$ 47 05 $= 107$, where the Tangent Line $e\,k$ cuts the Meridian $e\,d$, if there be no Error.

Then on the other Side of the Sub-stile make $e\,8 =$ Tangent 12 55 $= 23$, $e\,7 =$ Tangent 27 55 $= 53$, $e\,6 =$ Tangent 42 55 $= 93$, for that the Tangent Line $b\,k$ can contain no more Hours.

On the Sides of the □ make $e\,2 =$ Co-tangent 77 05 $= 46$, the Radius being 200, then is $e\,1 =$ Co-tangent, 62 05 $= 106$ $c\,12$, Co-tangent 47 05, $= 1{,}86$.

On the other Side of the □ make $b\,3 =$ Co-tangent 87 55 $=$ 72 $b\,4 =$ Co-tangent 72 55 $= 62$, $b\,5 =$ Co-tangent 57 55 $=$ 126 connect $b\,n =$ Co-sine 28 39 $= 175$, joyn $a\,n$ that is the Stile.

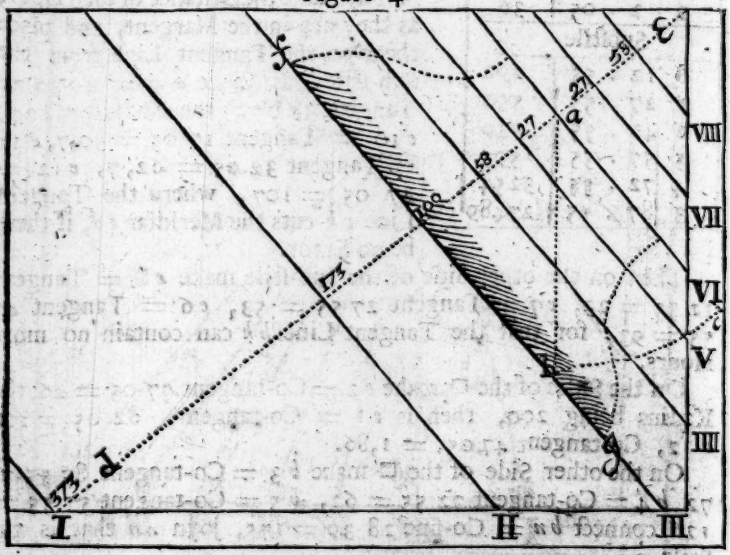
Observe, Here is 3 a Clock in the Morning marked, which is before Sun-rising, but if you continue it through the Centre, it will serve for the Afternoon.

Every 6 Hours have the same Distance on the Parallel Sides, as they have on the Tangent-Line, as $b\,4$ and $e\,10$, has 31, $b\,5$ and $e\,11$, has 63, $b\,6$ and $e\,12$, has 107; so $e\,6$ and $c\,12$ has 93; So $e\,7$ $c\,1$ has 53, $e\,8$ $c\,2$, has 23, $e\,9$ $c\,3$, has 3,6, for that 6 Hours is a Quadrant $=$ 90 deg. so that here is no occasion to take the Tangents of the Angle at the Pole, of above 6 or 8 Hours.

VII. *To make an Erect East or West Dyal, Lat.* 51° 15'.

If it be an Erect East or West Dyal, the Plane does decline 90° and by the Table, Page 15, you will find the Sub-stiles Distance = 38 45, the Stile to have no Altitude, and the Inclination of the Meridians = 90 *deg.*

Figure 4.



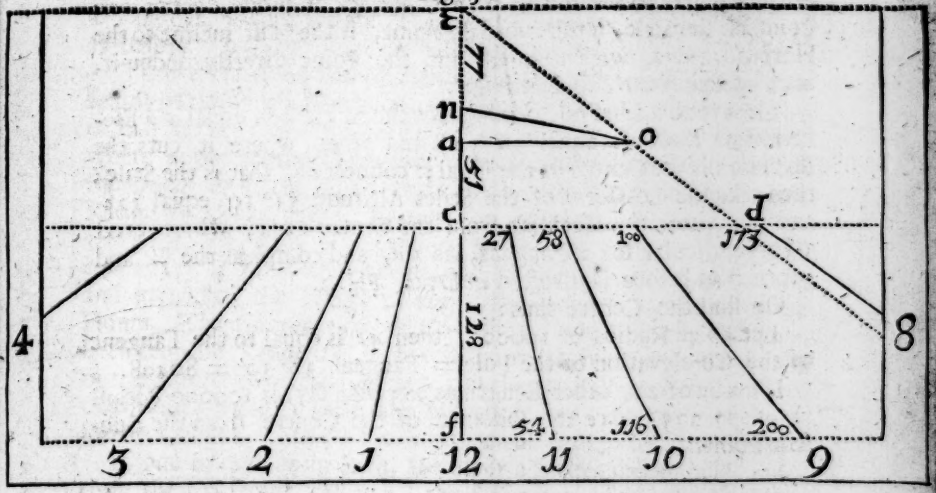
Angle at the Pole		
12	90	Tangent
1	75 =	373
2	60 =	173
3	45 =	100
4	30 =	57,7
5	15 =	26,7
6	0	Substile
7	15 =	26,7
8	30 =	57,7

First, Let fall *a b* perpendicular to the Horizon, then with the Chord of 60 draw an Arch of a Circle to the Point *c*, then take the Chord of the Substiles Distance 38° 45, from a Line of Chords or a Diagonal Scale equal 663 and place it from *b* to *c* to the Right-hand, because the Plane Declines Westward; joyn *a c*, that is the Sub-stile, which is the Hour of 6.

Though the Stile has no Angle of Altitude, yet we must place a Stile of a convenient length, about a third part of the Plane Paral-

lel

Fig. 5.



lel to the Sub-ſtile, to give the Shade; let the Stiles heighth be an Inch equal 100; to the Sub-ſtile apply *de*, Perpendicular for a Tangent, then find what Angle each Hour makes at the Pole with the Sub-ſtile, by comparing them with the Inclination of the Meridians; then take their Tangents in equal Parts, as in the Margent of the foregoing Page, and place them on the Tangent from the Sub-ſtile, through which Points draw Parallels to the Sub-ſtile for the Hours: the Line *fg* is the Hour of 3 and Stile.

VIII. *How to make an Horizontal Dyal, the End of a Staff, Poſt, or other Body giving the Shade, Lat.* 51° 10′.

You muſt conceive the end of the Staff or Poſt to be one Point of the Stile or Gnomen, and is called the Node, and any one Point of the Stile will give the Hour as true as the whole Stile, if the Hours are drawn long enough.

With a Square find the Point directly under the Node, which I call the Sub-ſtile Point, then meaſure the Diſtance from the Sub-ſtile Point of the Top of the Staff or Node, then find the Centre

as

as follows: Let *a e* be the Meridian, and *a o* the Staff ſtanding Perpendicular to the Meridian, *o* its Vertex or Node, being one Point of the Stile, *a* the Sub-ſtile Point, if the Staff incline to the Horizon as *n o*, with a Square find the Point directly under it, as *a*, which is the Sub-ſtile Point.

Then with a Line of Chords make the Angle *a o e* equal *c o* Elevation of the Pole equal 38° 50′, and at *e*, where it cuts the Meridian is the Centre of the Dyal; connect *e o*, that is the Stile; then take the Co-ſecant of the Stiles Altitude 51° 10′ equal 128, and place it on the Meridian from the Centre *e* to *c*, where erect a Perpendicular for a Tangent, as *c d*, and compleat the □ and proceed as before is directed in Dyal, Fig. 2.

Or find the Centre thus:

Let *a o* = Radius = 100000, then *a e* is equal to the Tangent of the Co-elevation of the Pole = Tangent 38° 50′ = 80498.

If *a o* be of any other Length, as 96; then ſay, as 100000 80498 :: 96 77 275 = *a e* the Diſtance of the Centre from the Sub-ſtile Point.

IX. *How to make a Dyal in the Ceiling by the Reflection of a Glaſs.*

A Reflecting Dyal (if the Ceiling of the Room on which it is to be made, and the Glaſs, be Parallel to the Horizon) is but an Horizontal Dyal reverſed; for that, according to the Opticks, the Sun, the Glaſs, and the Reflection are in a ſtreight Line, and the Angle of Reflection is equal to the Angle of Incidence, and the Glaſs you muſt conceive to be one Point of the Gnomon; with a little Water placed by the Glaſs, you may try by its Reflection whether the Glaſs ſtands Horizontally.

Firſt, you muſt meaſure the Diſtance between the Glaſs and the Ceiling, in Equal Parts, which ſuppoſe to be 4 Foot, or 48 Inches, or 96 half Inches; then make *a o* Equal to 96, and Perpendicular to the Meridian *a e*: then proceed as in the former Dyal, firſt finding the Centre, as is before directed.

But, foraſmuch as the Centre of the Dyal will not be in the Room (if the Window be Southward) you muſt draw the Hour Lines by two Tangents.

Having found *a e* = 77, the Radius being 100; ſo that *e* is the Centre of the Dyal; ſubſtract that out of the Co-ſecant of the

the Elevation of the Pole, *viz.* 128, rest 51, which take off the Scale, and place it from *a* to *c*, where erect a Perpendicular for a Tangent, where set the Tangent of 15, 30, 45, 60, and 75, as before.

Then from *c*, on the Meridian, set the Difference of the Co-secant of the Pole's Elevation. Of a longer Radius, either in Inches or half Inches; as suppose the Radius to be 2 Inches = 128 to *f*, where apply another Tangent, then for each Hour take double the Distances of the Equal Parts of the first Tangent Line: For Radius 100, *ec.* 128 :: Radius 200 *ef* 256, and 100, 200 :: *c* 1 26,8, *f* 1, 53,6 :: *c* 2 58, *f* 2 116 :: *c* 3 100, *f* 3 200, *c* 4 173, *f* 4 346; then the second Tangent being thus marked and joyned to the Marks of the first Tangent, you have the Hours. The difference of the Co-secants, which is the Distance between the Tangents, is found by multiplying the Co-secant in the difference of the Radius's in Inches or half Inches; as here, one Radius being 1 Inch, the other 2 Inches, multiply the Co-secant 128 in 1, then *c f* the difference of the Co-secant is 128.

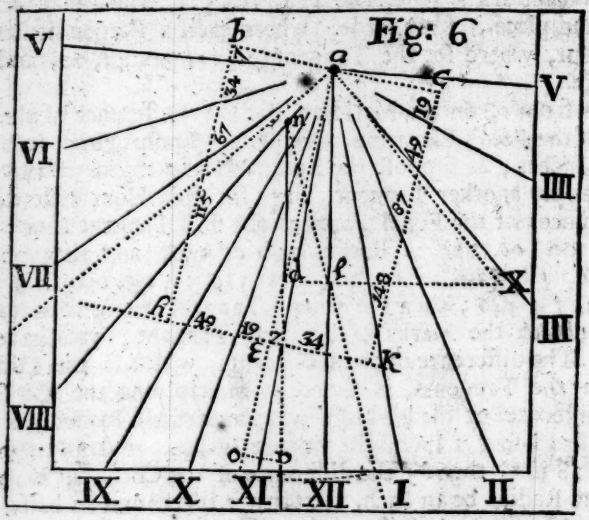
If one Radius be an Inch, the other 2 Inches and an half, multiply the Co-secant 128 in 3, the difference of the Radius's in half Inches the Product 384 half Inches is the Distance between *c* and *f*.

To find the Hours on the second Tangent, say; 2 : 5 :: 26,8, 67 :: 58 145 :: 100 250 :: 173 4325.

To save the labour of Multiplication, let the differences of the Radius's be equal to an Unite; *i. e.* to an Inch or half an Inch.

X. *How from the End of the Shadow of any thing, as a Pin, on an Upright Declining Plane, to make a Dyal, Lat.* 51°. 15'. *Let the Plane decline* 15° *Eastward, let the length of the Pin be* 9,5 *Inches.*

With a Square find the Point directly under the top of the Pin at *d*, which I call the Sub-stile Point; from that Point *d*, with a Line and Plummet let fall a Perpendicular to the Horizon, as *d p*; then by the Table Page 14, you will have for 15 *deg.* of Declination, the Substiles Distance to be 11° 44', the Stiles Heighth 37° 13', Inclin. Merid. 18° 58', make the Angle *p d o* = Substiles Distance



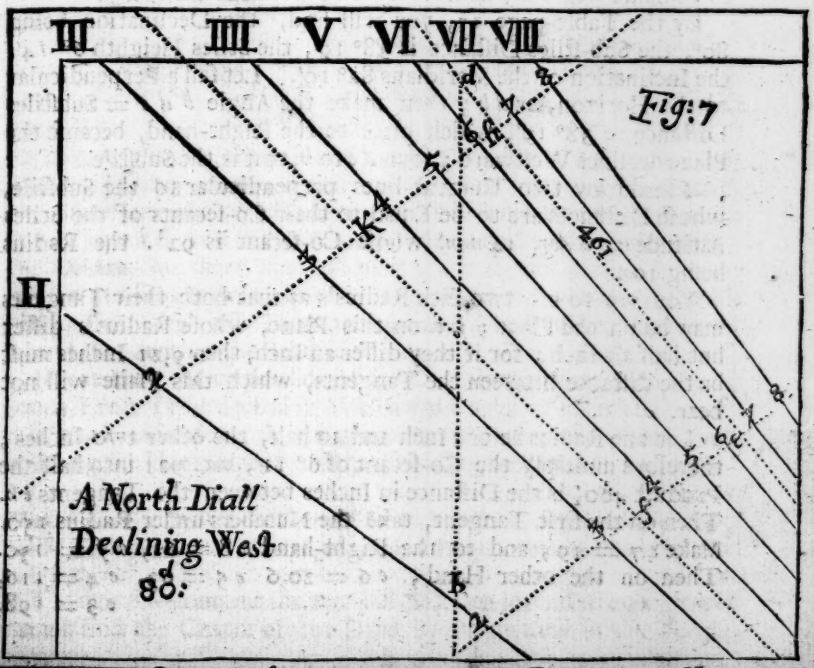
Distance = 11 44, and to the left-hand being the Plane, declines Eastward, continue *o d* and *p d*, make *d n* = to the length of the Pin = 9,5 Inches, and the Angle *d n l* = Declination of the Plane = 15°, and place it to the right-hand, because the Plane declines Eastward; make *d l*, perpendicular to *d p* for an Horizontal Line, and at *l*, where *l n* cuts the Line *d l*, is one Point of the Meridian; make *l a* parallel to *d n*, then *l a* is the Meridian; and at *a*, where it cuts the Substile, is the Centre of the Dyal. From the Centre *a*, set on the Substile 165, the Cosecant of the Stiles Heighth 37° 13′, the Radius being 100, to *e*, where apply a Perpendicular to the Sub-stile for a Tangent; and compleat the □ by making *a b*, *a c*, *e h*, *e k* = Sine 37° 13′ = 60, Radius being 100, connect *b h*, *c k*, then take the Tangents of the Angles that each Hour makes with the Sub-stile at the Pole, by comparing them with the Meridian, as it is in the Margent, and placing them on the Tangent from the Sub-stile, which joyn to the Centre, and you have the Hours of 9, 10, 11, 12, 1; and for the other Hours, take the Co-tangent of the Angles each Hour makes with the Pole, as *c* 4 = Cotangent 78 58 = 19, *c* 3 = Co-tan-

= Co-tangent 63 58 = 49, c 2 = Co-tangent 48 58 = 87. c 1 = Co-tangent 33 58 = 148. And on the other Side b 5 = Co-tangent 86 2 = 07, b 6 = Co-tangent 71 2 = 34, b 7 = Co-tangent 56, 2 = 67, b 8 = Co-tangent 41 2 = 115.

The Centre may likewise be thus found; make $l \times = l n$ and the Angle $l \times a$ = Elevation of the Pole = 51° 15′ and at a, where it cuts the Meridian, is the Centre of the Dyal.

Rad. 1000 37 13
Co-sine 796.
Sine 605
Co-secant 37 13 1653.
Secant 37 13 1255.

Hor. ∠ at Pole	Tang.
5 . 86 . 2 .	14421
6 . 71 . 2 .	2910
7 . 56 . 2 .	1484
8 . 41 . 2 .	870
9 . 26 . 2 .	488
10 . 11 . 2 .	195
Substile.	
11 . 3 . 58 .	69
12 . 18 . 58 .	343
1 . 33 . 58 .	673
2 . 48 . 58 .	1149
3 . 63 . 58 .	2047
4 . 78 . 58 .	5128



XI. *How to make an Upright declining Dyal in a Plane, where there is no Conveniency of having a Centre: A North Plane declining Westward* 80°. *Lat.* 51° 15′.

In the foregoing Scheme let half an Inch be = 100, and then the Numbers in the following Table are proper to it.

A North and South Plane of the same Declination are made after the same manner, and by placing the upper Side downward, a South Dyal will serve for a North Plane; but the Hours in the one are the Complement of 12 that they are in the other, as 3 In one, is 9 in the other; and 8 in one is 4 in the other, and some Hours are to be omitted, and others added. If a Dyal does decline about 70°, or upwards, and you place the Centre in the Dyal, the Hours near the Substile will be so close together, that the Hour-lines will be confounded: To prevent which, the Dyal is to be made without a Centre, by two Tangent-Lines.

By the Table-page 15, you will find, the Declination being 80°, the Sub-stiles Distance is 38° 19′, the Stiles Heighth 6° 14′, the Inclination of the Meridians 82° 10′. Let fall a Perpendicular to the Horizon, as *d b*; then make the Angle *b d e* = Substiles Distance = 38° 19′, which place to the Right-hand, because the Plane declines Westward; joyn *d e* to *h*, that is the Substile.

Then draw two Tangent-lines perpendicular to the Substile, whose Distances are to be Equal to their Co-secants of the Stiles Altitude = 6 *deg.* 14 *min.* whose Co-secant is 921. the Radius being 100.

You are to use two such Radius's as that both their Tangents may be on the Plane; *i. e.* on this Plane, whose Radius's differ but half an Inch; for if they differ an Inch, then 9,21 Inches must be the distance between the Tangents, which this Plane will not bear.

Let one Radius be one Inch and an half, the other two Inches; therefore multiply the Co-secant of 6° 11′, *viz.* 921 into half the Product 460½ is the Distance in Inches between the Tangents *e h*. Then on the first Tangent, take the Numbers under Radius 150. Make *e* 7 = 19, and to the Right-hand *e* 8 = 61, *e f* = 150. Then on the other Hand, *e* 6 = 20,6 *e* 5 = 63, *e* 4 = 116, *e* 3 = 198,

$e3 = 198$, $e2 = 368$. And on the second Tangent take the Numbers that are under Radius 200, as $h7 = 25$, $h8 = 81$, hk

Hor.	∠at Pole	Rad.	Rad.	Rad.
12	82 . 10	100	150	200
		Tan.	Tan.	Tan.
8	22 . 10	40. 7	61.	81
7	7 . 10	12. 5	18 1	25
	Substile			
6	7 . 50	13. 7	20,6	27
5	22 . 50	42.10	63.1	84
4	37 . 50	77. 6	116,5	155
3	52 . 50	131,9	19,8	264
2	67 . 50	245,4	36,8	491
1	82 . 50	796,3		

	Rad.	1000			
Stiles H. 6. 14.	Sine	108	10,8	16,2	21,6
	Co-sine	994	99,4	14,9	198,8
	Secant	1006	100,6	15,0	20,1
	Co-secant	9210	92,1	138,1	184,2

$= 20$. And on the other side of the Substile $h6 = 27$, $h5 = 84$, $h4 = 155$, $h3 = 264$, $h2 = 491$. Then joyn the Marks for the same Hours on each Tangent, and you have the Hours. And if you joyn kf, you have the Stile. You need not enquire after the Centre, for there is no use of it; but if you desire it, the Co-secant of the Stiles Heighth will give it you; *i.e.* it is 1381 half Inches from the first Tangent, and 18,62 half Inches from the second Tangent on the Substile.

If you turn the upper side of the Dyal downward, then it is a South Erect Dyal declining Westward 80 *deg.* Then you must new mark each Hour, by putting the Complement of 12. and omitting the Hours after Sun-set, and adding the Hour of 2 and 3.

How to make a Dyal on an Upright Declining Plane, a Pin, or any other thing giving the Shadow, the Centre being not on the Plane.

Though this contains the two last Sections joyned in one, yet we cannot find the Centre of the Dyal by the method in the Tenth

Section, it being not on the Plane; neither can we by the preceding Section draw the firſt Tangent at pleaſure, for that the Length of the Stile is given; but we muſt find the Diſtance of the Centre from the Subſtiles Point by Delineation or by Calculation, as by Sect. VIII. Then by comparing it with the Co-ſecants of the Stiles Altitude, the Diſtance of the firſt Tangent from the Subſtile Point is found.

As Latitude 51 *deg.* 15 *min.* the Pin being 14,5 Inches in length, the Declination of the Wall being 65 *deg.* Weſtward.

By the Table-Page 15, for 65 *deg.* of Declination, the Subſtiles Diſtance is 36 *deg.* 2 *min.* the Stiles Heighth 15 *deg.* 20 *min.* the Inclination of the Meridian is 70 *deg.* 1 *min.*

	Radius	1000
Stiles Heighth	Sine	264
15 *deg.* 20 *min.*	Co-ſine	964
	Secant	1034
	Co-ſecant	3781

By the preceding Rules, having drawn the Subſtile, the Centre of the Dyal being ſuppoſed not to be on the Plane; On ſome other Plane make a Rectangle Triangle, one of whoſe Legs is equal to the length of the Pin = 14,5 Inches; Then on the end of that Leg make an Angle equal to 74° 40′, then the other Leg is the Diſtance of the Centre from the Subſtile Point: Or by the Doctrin of Plain Triangles, as in the Eighth Section, thus,

As the Radius is to the Co-tangent of the Stiles Heighth,

So is the Length of the Pin to the Diſtance of the Centre from the Subſtiles Point.

Radius	10.000000
Co-tang. 15° 20′	10.561441
Pins Length 14,5.	1.161368
Centrual Diſtance 52,88	1.723309

Then taking the Difference between the Co-ſecant of the Stiles Heighth, 3781 Inches, and the Diſtance of the Centre from the Subſtile-Point, 52,88. *viz.* 15,07 Inches, and place it on the Subſtile Point towards the Centre, for that the Diſtance of the

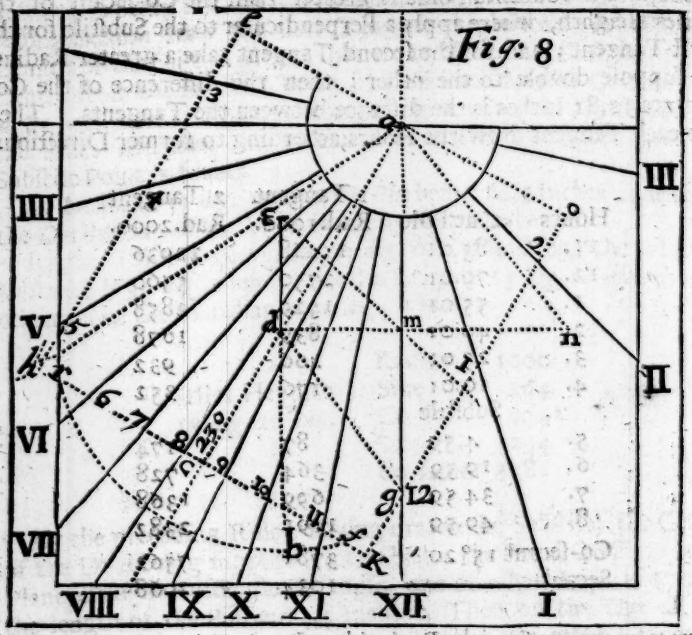
Centre

Centre of the Subſtile Point is greater than the Co-ſecant of the Stiles Heighth, where apply a Perpendicular to the Subſtile for the firſt Tangent; and for the ſecond Tangent take a greater Radius, as ſuppoſe double to the other; then the difference of the Co-ſecants 37,81 Inches is the diſtance between the Tangents. Then on each Tangent draw the Hours, according to former Directions.

Hours	∠ at Pole	1 Tangent Rad. 1000.	2 Tangent Rad. 2000.
11.	85.01	11468	22936
12.	70.01	2750	5500
1.	55.01	1429	2858
2.	40.01	839	1678
3.	25.01	466	932
4.	10.01	176	352
	Subſtile		
5.	4.59	87	174
6.	19.59	364	728
7.	34.59	699	1368
8.	49.59	1191	2382
Co-ſecant	15° 20′	3781	7562
Secant		1034	2068

How to make any Upright Dyal without knowing the Declination of the Plane, having the Latitude of the Place, and the Suns Shadow on the Plane at Noon. Lat. 51° 15′.

At a convenient place put an Iron Pin into the Plane, at leaſt 6 Inches in length, the Shadow of whoſe end ſhews the Hour, as *d e*, whoſe length is 6,8 Inches. With a Square find *d* the Point directly under it, which is the Subſtile Point. At 12 a Clock take notice of the end of the Shadow of the Node or Pin at *g*. Then through the the Subſtile Point *d*, with a Line and Plummet let fall a Perpendicular to the Horizon, as *d b*, and draw an Horizontal Line through *d*, as *d m*. Through *g* draw a Parallel to *d b*, which is the Meridian, and joyn *m* its Interſection with the Horizontal-line to the Node, as *m e*, then the Angle *d e m* is the Declination of the Plane = 46. On the Horizontal-line make *m n* = *m e*, and make the Angle *m n a* = Elevation of the Pole = 51°



= 51° 15′. Then at *a*, where it cuts the Meridian, is the Centre of the Dyal; connect *a d*, that is the Substile.

By the Table, Page 14. for 46° of Declination; the Substiles Distance is 30° = to the Angle *d a m*. The Stiles Heighth 25° 46′. The Inclination of the Meridian 53° 1′.

Then measure 230. the Co-secant of the Stiles Heighth 25° 46′. the Radius = 100 = Inch on the Substile from the Centre to *c*, where apply a Perpendicular for a Tangent, and another through the Centre; then compleat the □ by making *a l*, *a o*, *c h*, *c k* = 217. the Sine of 25 *deg.* 46 *min.* the Radius being 500 = 5 Inches. Then on the Tangent-line make *c* 9 = 8 *deg.* 1 *min.* = 14, and *c* 10 = Tangent 23 *deg.* 1 *min.* = 42, *c* 11 Tangent 38 *deg.* 1 *min.* = 78. And on the other side *c* 8 = Tangent 6 *deg.* 59 *min.* = 12, *c* 7 = Tangent 21 *deg.* 59 *min.* = 40. *c* 6 = Tangent 36 *deg.* 59 *min.* = 75.

And

Hours	∠ at Pole	Tangent
12 .	53 . 1	1327
11 .	38 . 1	782
10 .	23 . 1	424
9 .	8 . 1	141
	Substile	
8 .	6 . 59	122
7 .	21 . 59	417
6 .	36 . 59	753
5 .	51 . 59	1280
4 .	66 . 59	2354
3 .	81 . 59	7100
12 .	53 . 1	1330
1 .	68 . 1	2477
2 .	83 . 1	8144

Stiles Heighth 25 *deg.* 46 *min.*

Rad.	1000
Sine	434
Co-sine	900
Secant	1110
Co-secant	2300

And on the Sides of the □ make $l\,5 = \frac{1}{5}$ Co-tangent 51 *deg.* 59 *min.* = 391, $l\,4 = \frac{1}{4}$ Co-tang. 66. *deg.* 59 *min.* = 212, $l\,3 = \frac{1}{3}$ Co-tang. 81 *deg.* 59 *min.* = 70. On the other Side make $o\,12 = \frac{1}{12}$ Co-tang. 53 *deg.* 1 *min.* = 376, $o\,1 = \frac{1}{1}$ Cotang. 68 *deg.* 1 *min.* = 200, $o\,2 = \frac{1}{2}$ Cotang. 83 *deg.* 1 *min.* Joyn these Marks to the Centre, then you have the Hours.

How to make an Upright South Dyal, without knowing the Declination of the Plane, the Elevation of the Pole being known.

First place a Pin in the Plane at least six Inches long, as *d e*, almost upright, but a little inclining; with a Square find the Point directly under it, which is the Substile Point; and the end of the Pin which gives the Shade, the Stile Point. In the Morning, if the Sun shines on the Plane, observe the end of the Shadow of the Stile, as at *r*, and through it draw a Circle, whose Centre is the Substile Point. At some other time of the day mark where the end of the Stiles Shadow does come to some other Point of the same Circle, as at *f*, connect those two Points *f r*, then that Line *f r* will be a Tangent to the Substile. From the Substile Point *d* let fall a Perpendicular to that Tangent-line, as *c d*, which is the Substile. From *d* with a Line and Plummet, let fall a Perpendicular

cular to the Horizon, as *d b*. Then the Angle *b d c* is the Subſtiles Diſtance = 30 *deg.* which by the Table, Page 16, under the Subſtiles Diſtance, you will find the Declination of the Plane to be 46 *deg.* Inclination of the Meridians to be 53 *deg.* 01 *min.* and the Stiles Heighth 25 *deg.* 46 *min.* Then take its Co-ſecant 230, and place it on the Subſtile from *e* to *a*; then *a* is the Centre of the Dyal, and is to be made as Page 18.

If you deſire to find theſe things without the Table, by Calculation. Thus,

Co-ſine Subſtiles Diſtance 30 *deg.*	9.937531
Radius	10.000000
Sine Elevation Pole 51 *deg.* 15 *min.*	9.892036
Sine Stiles Heighth 25 *deg.* 47 *min.*	9.954499
Co-ſine Elevation Pole 51 *deg.* 15 *min.*	9.796521
Radius	10.000000
Sine Subſtiles Diſtance 30 *deg.*	9.698970
Sine Inclin. Merid. 53 *deg.* 1 *min.*	9.902449
Radius	10.000000
Tangent Elevation Pole 51 *deg.* 15 *min.*	10.095509
Tangent Subſtiles Diſtance 30 *deg.*	9.761434
Sine Declination Plane 46 *deg.*	9.856943

XII. *How to make a Nocturnal Dyal, to ſee what's a clock at Night by the Stars, or at Day by the Sun, when it ſhines bright, or enough to be diſcerned, but not clear to give a ſhadow.*

Though this ſort of Dyal be not common, yet it is very uſeful; For in any Room where there is a Caſement, and a Proſpect, you may make it, though it does not reſpect the South: I will firſt deſcribe it, and ſhew how to make and uſe it.

In the Caſement of a Window, a Ring muſt be fixed or a Scotch filed, or the Angle of the Caſement, or ſome othe Mark in the Caſement, or Bar of the Window, and a String ſomewhere in the Ceiling, or in the Wall above the Window in the Room,

Room, to be fixed according to Art, in a South Plane, or somewhere below, in a North Plane: In the inner Side of the Window, by the Motion of the String, the Plane of the Dyal is described, as if it were of Boards faſtned to the Window, and the Hours are to be placed on the Wall on the inſide of the Window.

How to uſe the Nocturnal Dyal.

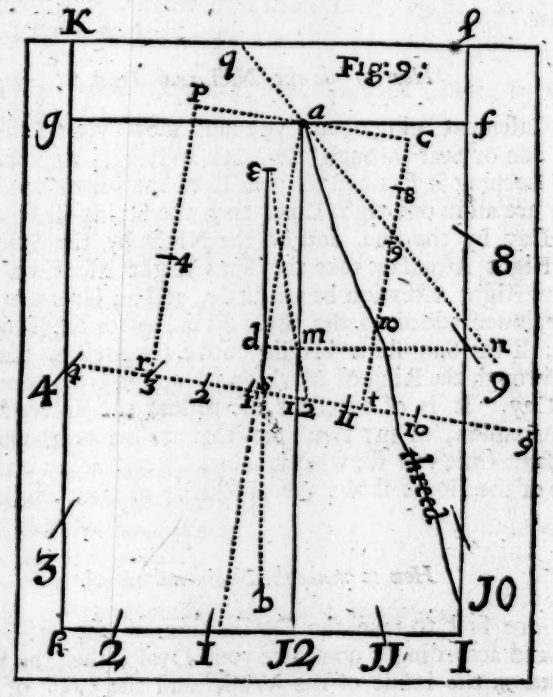
The Caſement being open, you muſt move your Body till you ſee the Sun or Star through the Mark or Ring, then move your String, keeping it ſtrained till your Eye, the Sun or Star, and the String, are all in one right Line, then the String ſhews the Hour of the Day by the Sun, and of the Night by the Star. From whoſe Right Aſcenſion take the Suns Right Aſcenſion (But if the Stars Right Aſcenſion be too little, add 24 Hours to it) then the remainder added to the Stars Hour, gives the Time of the Night: If the Sun ſhine bright, move the String, till the Sun ſhines through the Ring or Mark on it, and it gives you the Time of the Day. It is of excellent uſe to find the difference of the Right Aſcenſion, of any two Stars that are before the Dyal; for at the ſame time you ſee what is aclock by each Star; and the difference of the Hours ſhews the difference of their Right Aſcenſions.

How to make the Nocturnal Dyal.

You are firſt to take the Declination of the out-ſide of the Wall, and accordingly nominate your Dyal; then the String being fixed on the inſide of the Wall, and the Wall of an equal Thickneſs, the Motion of the String does deſcribe a Plane even with the inſide of the Window, and parallel to the outſide; ſo that you are to meaſure in equal parts, the neareſt Diſtance of the Ring or Scotch, from the Plane, deſcribed by the Motion of the String, and then the Dyal is the ſame with that Numb. X. and ſo to be made.

If the Dyal be on a North Plane, then inſtead of a String, you may uſe a Ruler, for that the Centre of the Dyal will be below.

There is a South Window, declining weſtward, 10°, Lat. 51° 10′, where a Ring is fixt to the middle of the Caſement, whoſe Diſtance from the Sub-ſtile Point, on the Plane, deſcribed by the String, is 10,8 Inches.



We muſt conceive the Plane *k l i h*, to be the inſide of the Window, *g f* a piece of Board faſtned to receive the Centre of the Dyal, to which the String is to be fixed, *e* to be the Ring in the Caſement, and the Node or one Point of the Stile, whoſe Diſtance from *d*, the Sub-ſtile Point is 10,8 Inches; but foraſmuch as it cannot be drawn on the Window, let it be done on a Table-Board, deſcribing the Plane *k l i h*, and on each Side its true Diſtance from *d*, the Sub-ſtile Point; let *e d b* be Perpendicular

to

Ho.	∠ at Pole.	Tang
8	72 . 44	3217
9	57 . 44	1584
10	42 . 44	924
11	27 . 44	525
12	12 . 44	220
	Substile.	
1	2 . 16	40
2	17 . 16	311
3	32 . 16	631
4	41 . 16	1082
5	62 . 16	1904

	Radius	1000
Stiles	Sine	617
Heighth	Co-sine	787
	Secant	1271
38 7.	Co-secant	1620

to the Horizon, and $de = 10{,}8$ Inches, make dmn perpendicular to de, and the Angle dem = Declination of the Plane = 10°, which place to the Right-hand, becauſe the Window declines Weſtward; at m where it cuts the Horizontal Line dn, is one Point of the Meridian, where draw a parallel Line to db, for the Meridian; then on the Horizontal Line make $mn = me$, and the Angle mna = Elevation of the Pole = 51° 10′; and at a, where it cuts the Meridian, is the Centre of the Dyal, where faſten a piece of Board, as gf: To take the Centre, joyn ad, that is the Sub-ſtile; then by the Table-page 16, for 10° declination the Sub-ſtiles Diſtance is 7° 58′, Stiles heighth 38° 7′, inclination of the Meridians 12° 45′: Then take the Co-ſecant of the Stiles heighth 38° 7′ = 162, the Radius being 100, and place it from the Centre on the Sub-ſtile from a to s, where draw a Tangent, as rst.

Then take the equal Parts that each Hour makes on the Tangent with the Sub-ſtile, as in the Margent before the Dyal, and place theſe Parts on the Tangent from the Sub-ſtile, and joyn them to the Centre, and the Dyal is finiſhed. Here is no occaſion of finiſhing the □, for that the Tangent rst continued, will contain as many Hours as will ſhine on the Plane; but if you are deſirous to deſcribe the □, make ac, ap, sr, st = Sine 38° 7′ = 62 joyn pa, ct, your □ is finiſhed.

Obſerve, In theſe Nocturnal Dyals, the Hours to be differing from other Upright Dyals, for the Morning-Hours are on the

Right-hand, and the Evening-hours on the Left-hand, for that we do ſtand on the North-ſide of the Dyal, as in Chap. 3.

If the Piece of Board *g f* had not been faſtned to have cut the Centre, then the String might have been fixed to the Ceiling at *q*, where the Line *n a* being continued, touches the Ceiling; but then the Plane is not an upright, but an inclining Plane, yet the Dyal will be true; neither is it properly a Plane to the Hours that are on the Sides of the Window, but the Motion of the String deſcribes an irregular Body, yet the Dyal will go true.

The Line *n a q*, though it is here to the Right-hand, yet on the Dyal it is on the Meridian: If the Thred be fixt above, and drawn through the Ring and ſtrained, it repreſents the Stile, and will make an Angle with the Quadrant equal to the Elevation of the Pole = 51° 10′.

XV. *How to make any Upright, Declining Dyal for any Latitude, without the Table-page* 15, *Lat.* 51° 15′, *declining Eaſtward*, 25°.

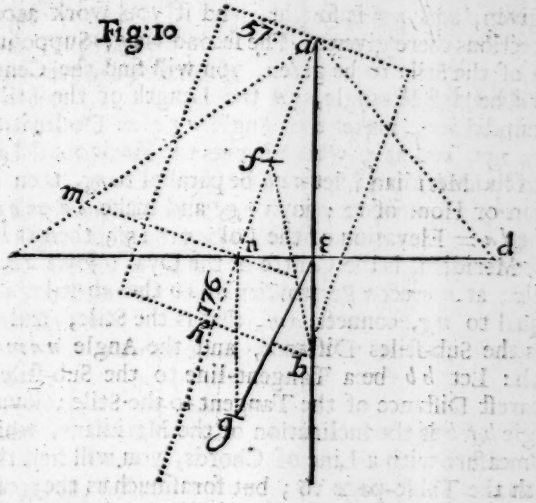
Let *a* be the Centre of the Dyal; *a b* the Meridian, which cut at right Angles with the Line *n e d*; make the Angle *e a d* equal Co-elevation of the Pole = 38° 45′; then make the Angle *b e g* = Declination of the Plane = 25°, and place

the Angle to the { left- / right- } hand, the Plane declining { Eaſtward. / Weſtward. }

make *e g* = *e d*, and draw *g n* parallel to *a b* the Meridian, joyn *a n*, that is the Sub-ſtile, to which at *n* apply *m n* Perpendicular to the Sub-ſtile *a n*, let *m n* be = *n 9*, connect *a m* that is the Stile. So that the Angle *n a e* is the Sub-ſtiles Diſtance, and the Angle *n a m* is the Stiles Heighth = 34° 33, take its Co-ſecant 176, and place it from the Centre *a* on the Sub-ſtile to *h*, where, at Right-angles to the Sub-ſtile apply a Tangent, and mark the Hours as uſual. To find the Inclination of the Meridians, from *h* take its neareſt Diſtance to the Stile, and place it on the Sub-ſtile from *h* to *f*, connect *f b* where the Tangent cuts the Meridian, then the Angle *h f b* is the Inclination of the Meridians.

If

Fig. 10



Hor.	∠at Pole	Tang.
4	89 . 7	64858
5	74 . 7	3514
6	59 . 7	1672
7	44 . 7	969
8	29 . 7	557
9	14 . 7	251
	Substile	
10	0 . 53	15
11	15 . 53	284
12	30 . 53	598
1	45 . 53	1031
2	60 . 53	1795
3	75 . 53	3976

	Radius	1000
Stiles	Sine	567
Heighth	Co-sine	823
	Secant	1214
34°. 33′.	Co-secant	1763

If you compare this Method with the 10 Section, you will find them both to be done on the same ground; for in both, the Elevation of the Pole, and the Declination of the Plane are given, and in that the length of the Stile from the Plane in equal Parts, *viz.* *ng* is given, and the Distance of the Centre on the Meridian

dian from the Horizontal Line, *viz.* *e a* is sought after; in this *e a* is given, and *n g* is sought, and if you work according to the Directions there given. The second way: Supposing *n g* the Length of the Stile to be given, you will find the Centre *a*: As let *g n d* be a Rect-angle, *g n* the Length of the Stile, *n d* the Horizontal-Line, make the Angle *n g e* = Declination of the Wall = 25°, and at *e*, where it cuts the Horizontal-Line, is one Point of the Meridian; let *a e b* be parallel to *n g*, then *a e b* is the Meridian or Hour of 12; joyn *e g*, and make *e d* = *e g*, and the Angle *e d a* = Elevation of the Pole 51° 15′; then at *a*, where it cuts the Meridian, is the Centre of the Dyal; joyn *a n*, that is the Sub-stile; at *n* erect a Perpendicular to the Sub-stile *a n*, as *n m*, and equal to *n g*, connect *a m*, that is the Stile, and the Angle *n a e* is the Sub-stiles Distance, and the Angle *n a m* the Stiles Heighth: Let *h b* be a Tangent-line to the Sub-stile, and *h f* the neareſt Diſtance of the Tangent to the Stile; joyn *f b*, then the Angle *h f b* is the Inclination of the Meridians, which Angles, if you meaſure with a Line of Chords, you will find them agreeing with the Table-page 16; but foraſmuch as they cannot be ſo exactly meaſured as calculated, I will give you the Calculation by plain Triangles.

1 △, *a e d* given *a e* = 275, ∠ *e a d*, Co-elevation 38 45 to find *d e* Radius.

a e	275 044	2.439263
Tangent *e a d*	38 45	9.904491
d e	220 6	2.343754

2 △, *e n g* given, *g e* = *d e* 220 6, ∠ *e g n* = Declination 25, to find *n e g n* Radius.

g e (*d e*)	220 6	2.343754
Sine *e g n*	25	9,625948
n e	93 26	1.969702
Sine *g e n*	65	9.957276
g n	209	2.301030

3 △, *e a n* given, *a e* 275, *n e* 93 26, to *e a n a n*.

e a	275	2.439263
e n	93 26	1.969702 Subſt.d.
Rad.		10.
Tan. *e a n*	18 44	9.530439

4 Sine *e a n* 18 44 9.506727
e n 93 26 1.969702
Sine *a e n* 90 10.000000
a n 290 4 2.462975

5 △,

5 △, *a n m* given, *a n* *n m* to find *n a m*.

a n 2904	2.462975
n m 200	2.301030 Stiles h.
Rad.	10.
Tang. *n a m* 3433	9.838055

6 △, *a h b* given, *a h* = Cosecant, Stiles heighth 34 33 = 352,6, Radius = 200, < *h a b* = Sub-stiles Distance, 1844, to find *h b*.

Radius	10.0000000
a h 3526	2.547282
Tang. *h a b* 18 44	9.530439
h b 1196	2.077721

7 △, *f h b* given, *h f* = *d e* = Radius *b h* 1196, to find < *h f b*.

h f 200	2.301030
h b 1196	2.077721
Radius	10.
Tang. *h f b* 30 53	9.776691

Inclination of the Meridian agreeing with the Table-page 15.

If *n g*, the Heighth of the Stile in equal parts be given.

1 △, *e n g* given, *n g* 200, < *e n g*, Declination of the Plane 25, to find *n e* *g e*.

Sine *n e g* 65	9.957276
n g 200	2.301030
Sine *e g n* 25	9.625948
n e 93 26	1.969702
Sine *e n g* 90.	10.000000
g e 2206	2.343754

2 △, *a e d* given, *d e* (*g e*) 220,6, < *a d e* = Elevation of the Pole 51 15 to find *a e*.

Radius	10.
d e (*g e*) 220,6	2.343754
Tangent *a d e* 51 15	10.095509
a e 275 044	2.439263

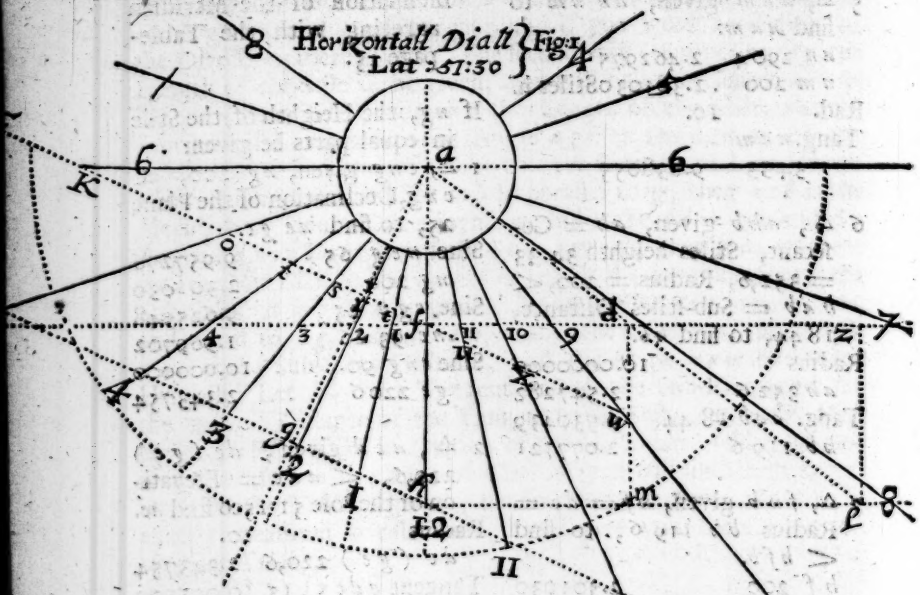
Then proceed as before.

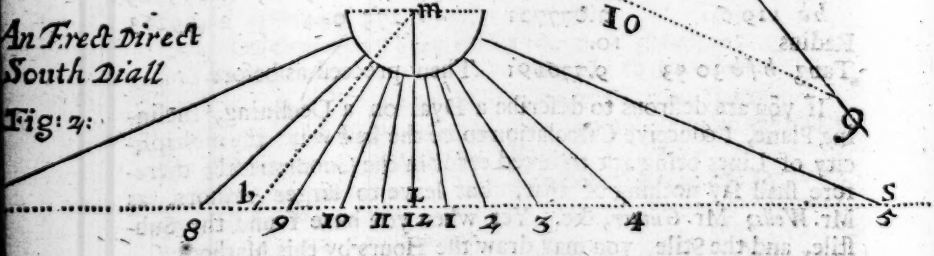
If you are desirous to describe a Dyal on a Declining, Inclining Plane, I conceive Calculation to be the best way, the multiplicity of Lines being apt to breed error in the Geometrical; therefore shall say nothing of that, but leave to larger Authors, as Mr. *Wells*, Mr. *Gunter*, &c. Yet when you have found the Substile, and the Stile, you may draw the Hours by this Method.

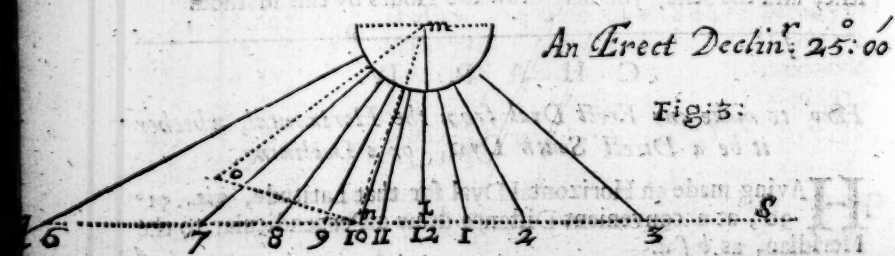
CHAP. IV.

How to make any Erect Dyal from the Horizontal, whether it be a Direct South Dyal, or a Declining.

HAving made an Horizontal Dyal for that Latitude, *viz.* 51° 30', at a convenient Distance draw a Perpendicular to the Meridian, as *b f d*.


Horizontall Diall
Lat: 51:30
Fig: 1
8
4
5
6
a
6
K
o
4
3
2
1
f
11
10
9
d
z
7
4
3
g
2
1
h
12
11
10
m
l
8
9


An Erect Direct
South Diall
Fig: 2:
m
b
L
S
8
9
10
11
12
1
2
3
4
5


An Erect Declin.r 25°:00′
Fig: 3:
m
o
n
L
S
6
7
8
9
10
11
12
1
2
3

I. *A Direct South Dyal.*

Then, if you are to delineate a Direct South Dyal, on your Plane draw two Lines $q < s$, $< m$ cutting the other at Right Angles at $<$, them $< m$ is the Meridian; mark the same Divisions on the Line $q < s$, as there are on $f d$, *i. e.* < 1, $< 11 = f 11$, $f 1$, < 2, < 10, $f 10$, $f 2$, < 3, $< 9 = f 9$, $f 3$, < 4, $< 6 = f 8$, $f 4 < 5$, $< 7 = f 7 f 5$, then on the Meridian make $< m = f d$, then m is the Centre of the Dyal, which connect to the Marks before made, you have the Hours from 5 to 7: Through the Centre draw a Perpendicular to the Meridian, that is the Hour of 6; then on this Dyal place $< b =$ to $f a$ on the Horizontal and joyn $m b$, you have the Stile: You must remember to mark your Hours right, the Hours on your Right-hand are Afternoon, and hours on the Left-hand Morning Hours.

II. *An Erect East or West Dyal.*

Make an Angle with the Meridian $= < f d a$, on the Horizontal Dyal, and

if it { East / West } be an $<$ Place the $<$ to the { Left- / Right } hand, as $< m b$

then $m b$, which on the South Dyal is the Stile, here it is the Hour of 6 and Substile, which crosses at Right-Angles, that is the Tangent; then take those Distances that are on the Line $f d$ in the Horizontal, as $f 1$, $f 2$, $f 3$, and place them on the Tangent-Line from the Sub-stile on both Sides of it, where draw Parallel-Lines to the Sub-stile, you have the Hours; those next the Sub-stile are the Hours of 5 and 7, and the second from the Sub-stile are the Hours of 4 and 8, *&c.* This Stile having no Angle of Altitude, must be equal in length to the Hour of $3 = f 3$, on the Horizontals.

III. *An Erect Dyal, declining Eastward 25°, Lat. 51° 30′.*

Having on the Horizontal Dyal drawn $b f d$, at f, where it cuts the Meridian make the Angle $b f c =$ to the Declination

of the Plane = 25°; and to the Left-hand, being it declines Eastward (to the Right, if it had declined Westward).

Then, having erected $< m$ perpendicular to the Line $q < s$, mark the same Divisions on the Lines $q < s$ on the Declination as there are on the Line $k f w$, on the Horizontal, beginning at the Meridian, as make < 11 on the Decliner $= f e$ on the Horizontal $< 10 = f r$, $< 10 = f s$, $< 8 = f t$, $< 7 = f u$, $< 6 = f k$, and on the other Side of the Meridian $< 1 = f h$, $< 2 = f x$, and $< 3 = f w$. On the Horizontal, from the Centre a, let fall a Perpendicular to the Line $k w$, as $a i$, then $f 1$ is the Sub-stiles Distance, to which make $< n$ on the Decliner equal, and make $< m$ on the Meridian of the Decliner $= f a$ on the Horizontal, joyn $m n$, that is the Sub-stile, and m is the Centre of the Dyal, which connect to the former Mark, and you have the Hours; at n, where the Substile cuts the Horizontal Line $q s$, erect the Perpendicular $n o$, and make $n o =$ to $i a$ on the Horizontal, which is the Stiles Heighth, joyn $m o$ that is the Stile.

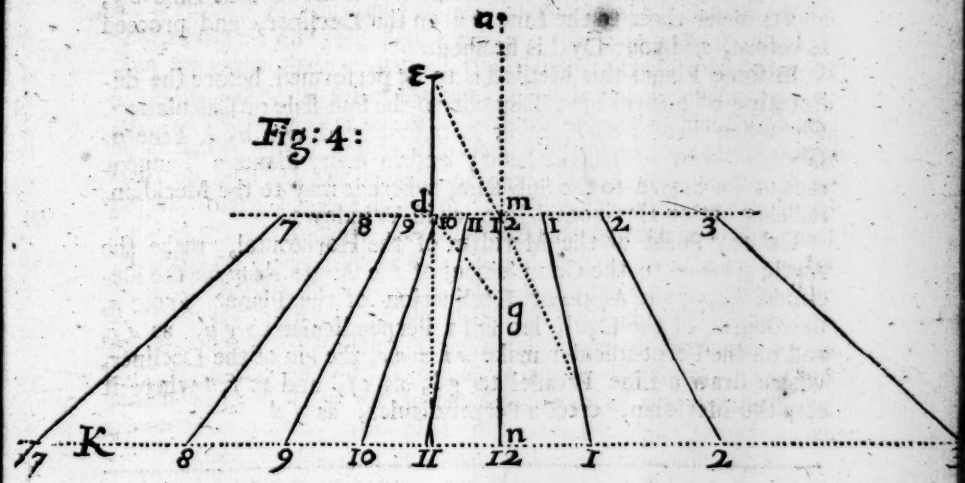
IV. *How to make any Upright Dyal, without taking the Declination of the Wall, having the Latitude of the Place, and the Shade of the Sun at Noon*; i.e. *the Dyal N° 12, though it has no Centre by the Horizontal.*

Let $d e$ be a Pin fixt into the Wall, d the Sub-stile-Point, e the Node, or the top of the Pin, $d g$ its Shade at Noon, through g let fall the Perpendicular to the Horizon, as $m g m$, which is $m g n$ the Meridian, through the Sub-stile-Point d draw an Horizontal Line, as $d m$, joyn $e m$; then the Angle $d e m$ is the Declination of the Plane a; then from the Horizontal take the Division each Hour does make on the Line $c f w$ from the Meridian, and place them on the Decliner from m on the Horizontal Line $m d$, as it is in the Margent.

Then make on the Meridian of the Declination $m a = f d$, then a is the Centre of the Dyal: But here being supposed to be none on the Plane, at a convenient Distance on the Horizontal, draw another Line parallel to $f d$, as $h i$, and at h, where it cuts the Meridian a Parallel to the Line $c f$; then on the Horizontal take the Difference between $h l$ and $f d$, which is $m l$ or

$d z$,

Fig: 4:



d e and place it on the Meridian of the Declination from *m* to *u*, where draw a Parallel to *m d*, as *k n*, then take the Distance each Hour does make on the Horizontal *a n* the Line *g h* from the Meridian, and place it on the Meridian of the Decliner from *n*, as in the Margent; then connect the Marks on both Parallels of the same Hours, and your Dyal is finished.	*m*	11	*f e*	
	m	10	*f r*	
	m	9	*f s*	
	m	8	*f t*	
	m	7	*f o*	
	m	6	*f k*	
	m	1	*f v*	
	m	2	*f x*	
Here is no need of finding the Substile, for *d* is one Point of it; nor the Stiles Heighth, for it is given, it being = *i*, *a* = *d e*, and *e* is the Node, or one Point of the Stile.	*m*	3	*f w*	=
	n	11	*h*	1
	n	10	*h*	2
If the two Parallel Lines *d m*, *k n* be first drawn on the Decliner, then take the Distance on the Meridian between *n* and *m*, and place it on the Horizontal, on the Line *f d*, from *d* to *z*, where draw a Parallel to the Meridian, as *z l*; and at *l*, where it cuts the Stile, let fall a Perpendicular to the Meridian, as *l h*, and at *h*, where it cuts	*n*	9	*h*	3
	n	8	*h*	4
	n	7	*h*	5
	n	1	*h*	11
	n	2	*h*	10
	n	3	*h*	9

the Meridian, draw a Parallel to *c f* as *g h*; then, as is before directed, take the Diſtances each hour does make on that Line *h g*, and transfer them to the Line *k n* on the Decliner, and proceed as before, and your Dyal is finiſhed.

In ſome Planes this Method is to be performed before the delineating of a Dyal by a Tangent to the Sub-ſtile or Calculation, for when there is no Centre, it cannot be drawn by a Line of Chords but by a Tangent-Line; and in many Planes a Tangent cannot be drawn to the Sub-ſtile, where it may to the Meridian, it being but an Horizontal-line, as in Windows.

On any Point of the Meridian of the Horizontal, make the Angle *g h a* = to the Complement of the Angle *d e m* on the Decliner, being the Angle of Declination of the Plane. From *a*, the Centre of the Dyal, let fall a Perpendicular to *g h*, as *a g*, and on the Perpendicular make *a i* = *d e*, the Pin of the Decliner, where draw a Line Parallel to *g h*, as *c f*, and at *f*, where it cuts the Meridian, erect a Perpendicular, as *f d*.

CHAP. V.

How to deſcribe the Tropiques and other Circles of Declination of the Sun on a Plain Dyal, the elevation of the Pole, and the Sun's Declination being given, firſt Geometrically, then Arithmetically.

IF the Stiles Heighth on the Plane be leſs than the Complement of the Sun's Declination, the end of the Shade of the Stile does deſcribe an Hyperbole; if equal a Parabole, if greater an Elipſe; I will give you an Example of Horizontal Dyals of each ſort, though the firſt and ſecond never happens in *England*, but on Inclining or Reclining Planes.

1. An Horizontal Dyal, the elevation of the Pole being 66° 30′, At *London*; it being a North Plane inclining to the Horizon 15°.

2. An Horizontal Dyal for the Latitude of 80°; or at *London*, a North Plane inclining to the Horizon 28° 30′.

3. An Horizontal Dyal for *London*, whoſe Latitude is 51° 30′.

Draw 2 Lines, interſecting at Right Angles, as *i d* 6, 1 6.

Let

Let *i*, the Point of Interſection, be the Centre of the Dyal, *i d* the Meridian or Hour of 12, make the Angle *b i y* = Elevation of the Pole = 66° 30′, Fig. 1 = 80°, Fig. 2 = 51° 30′, Fig. 3; then *b i* is the Stile or Axe of the World, from any Point of the Stile, as *b*, let fall a Perpendicular to the Sub-ſtile, as *b y*, which is the Length of the Stile from the Plane, then at the Node, or Vertex *b*, make the Angles *i b d*, *i b g*, equal to the Complement of the Sun's Declination 66° 30′; when he is in the Tropiques, make *b g* = *b d*, joyn *g d*, then the Iſoſceles Triangle *g b d* is the Triangle of the Cone biſected through it's Axe, and the Stile *b i* is the Axe of the World and of the Cone, and *d* where it cuts the Meridian is the Vertex of the Curve, when the Sun is in the Tropique of ♋, which in the Firſt Figure you may perceive to be a Parabole, for that the Side of the Cone *b g* is parallel to the Axe *i d*: From *d*, with the Diſtance *g d* cut *g b* continued in *k*, then *g k* is the Parameter of the Parabole, of which take ¼ part, and place it on the Meridian or Axe from *d* to *p*, then *p* is the Pole or Focus of the ſaid Parabole.

The 2 Figure is an Elipſe, for that *g b*, the Side of the Cone continued, does cut the Axe in *w*, below the Vertex *d*.

The 3 Figure is an Hyperbole, for that *g b* the Side of the Cone continued, cuts the Axe in *w*, above the Vertex *d*, and ſo is the Vertex of the oppoſite Hyperbole, when the Sun is in the the Tropique of ♑.

In both theſe *d w* is the Tranſverſe Axe, which if you biſect in *a*, then *a* is the Centre of the ſaid Curve's, and *g w* is the Focal Diſtance, which if you biſect in *t*, and put *t w* from the Centre *a*, both ways on the Axe, as *a p*, *a m*, you have *p m*, the Poles or Focuſſes of the ſaid Sections.

Then on each of theſe Figures apply ſeveral Perpendiculars to the Axe at the Vertexes, Poles, Centres, and other Points of the Axe; as *d q*, *p u*, *a x*, *n s*, *n s*, make at the Vertex *d q* = *d p*; and in the Parabole, Fig. 1, *p u* = 2 *d p*, connect *q u*, that is the Focal Tangent, which is a Line that touches the Curve, where an Ordinate applyed to the Axe at the Focus cuts it.

And in the other two Curves, at the Centre *a*, make *a x* = *a d*, which in the Hyperbole, Fig. 3, place on the other Side of the Axe; joyn *x q*, that is the Focal Tangent of theſe two Sections.

Then

Then on each of theſe Perpendiculars, with the neareſt Diſtance between the Axe and the Focal Tangent *n s* from the Pole *p*, draw an Arch, cutting the ſaid Perpendicular in *r*, then all theſe *r*'s are Points of the ſaid Curve, which if you joyn with a ſteady hand, you have the Section deſired.

In all theſe Sections, the Ordinate *h u*, which paſſes through the Focus, is the Parameter or *latus rectum* of the Axe, Figure 3; the oppoſite Hyperboles are the ſame, having the ſame Axes and Centre, and the ſame Focal Diſtance.

In the Hyperbole, from the Centre *a*, with the Semi-focal Diſtance, cut the Tangent of the Axe in *f*, then *d f* is the Length of the Semi-conjugate Axe, which if you connect to the Centre, and continue, you have one of the Aſymptotes, that is a Line, to which the Hyperbole does grow nearer and nearer, but never touch.

I do make but one half of the Dyal, leſt too many Lines ſhould make a Confuſion.

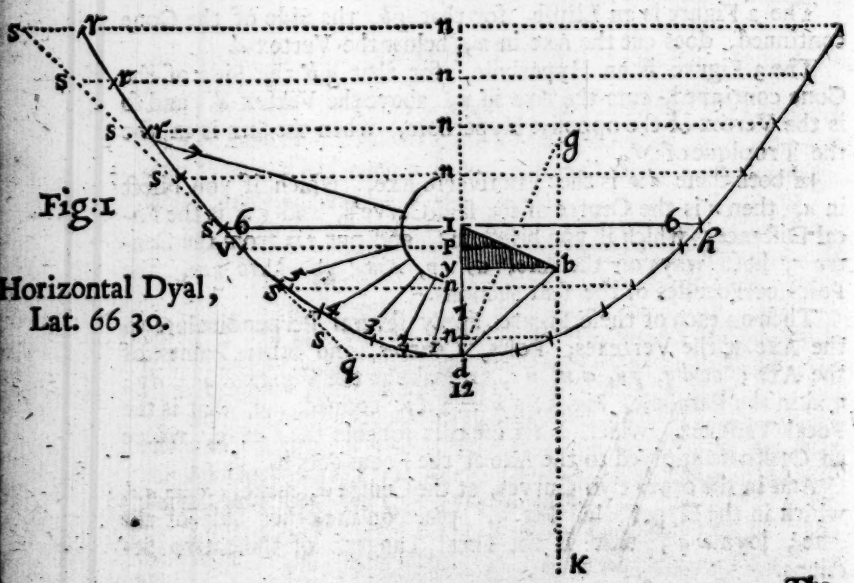


Fig: 1

Horizontal Dyal, Lat. 66 30.

The

The Angles the Hours make with the Meridian in the Parabole by Calculation, Lat. 66 30.

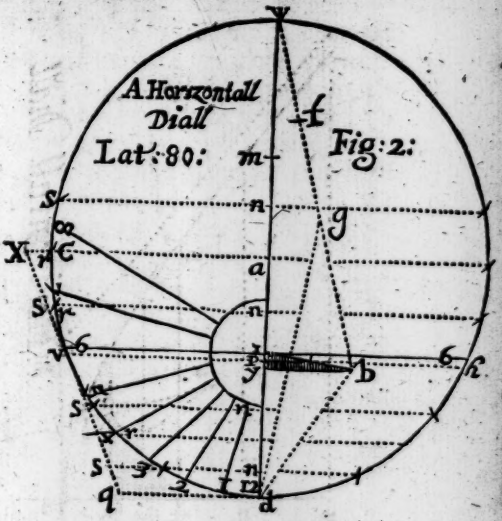
1	13	48
2	27	54
3	42	31
4	57	49
5	73	42

The Angles the Hours make with the Meridian in the Hyperbole, by calculation, Lat. 51° 30.

1	11	50
2	24	19
3	38	3
4	53	35
5	71	06

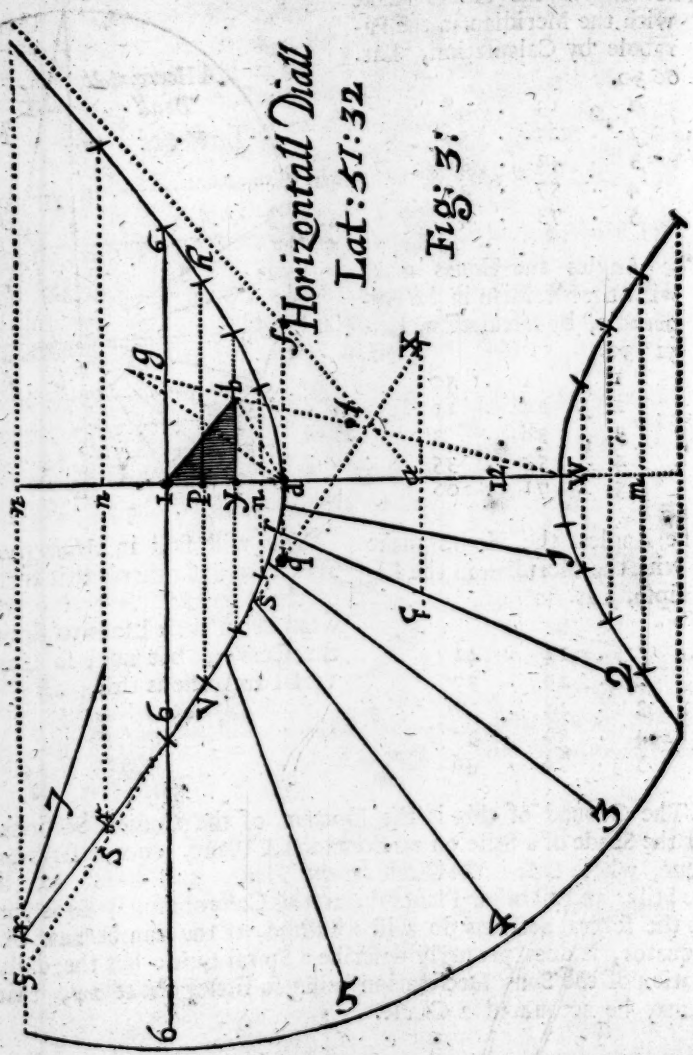
The Angles the Hours make with the Meridian in the Ellipse, Lat. 80°.

1	14	42
2	29	37
3	44	34
4	59	37
5	74	46



You will find in *Midorgius*, *S. Vincent*, and others, that treat of the Conicks, several other ways from these Lines to draw the Curves, but none so Universal and easie as this.

The Ground of this is the Doctrin of the Conick Sections, for the Shade of a Stile on an Equinoctial Plane, does describe a Cone, whose Base is the Circle on the Plane, and whose Axe is the Stile, and all other Planes do cut this Cone obliquely, so as you see the several Sections do arise: Indeed, if the Sun be near the Æquator, it does properly describe a Spiral Line; but the daily Motion of the Suns Declination being so little, about 24′, that it may be accounted a Circle.



If

If you desire these Lines in Numbers, suppose by the Stiles Heighth in equal Parts to be 100000, then in the Parabole, Fig. 1, $y\,d$ is the Tangent of the Angle $y\,b\,d = 43°$, the Complement of the Sun's Meridional Altitude, when he is in the Tropique of ♋, Lat. 66° 30′ = 93251, and $b\,d$ is the Secant of the same Angle = 136732, then by Trigonometry you may find $g\,d$, the Base of the Cone, and $g\,k$ the Parameter: Thus,

Sine $b\,g\,d$ 23 30	9.600700	and for	$b\,d$ 136732	5.135873	
$b\,d$ 136732	5.135873	that $b\,d$	$g\,d$ 250786	5.399300	
Sine $g\,b\,d$ 133 047	9.864127	$g\,d$ $g\,k$	$g\,d$ 250786	5.399300	
$g\,d$ 250786	5.399300	∴ ∴	$g\,k$ 459970	5.662727	

The Parameter.

In an Elipse, Lat. 80°, Fig. 2.

$w\,d$ the summ of the Tangents, $w\,g$ the difference of the Secants — of $y\,b\,w$ and $y\,b\,d$ the Complement of the Sun's Meridional Altitude, when he is in the Tropique of ♋, is the — Transverse Axe, Focal, Distance.

$w\,y$ Tang. $y\,b\,w$	76 30	4165530	$w\,b$ Secant $y\,b\,w$	76 30	428366
$y\,d$ Tang. $y\,b\,d$	56 30	151083	— $b\,d$ Secant $y\,b\,d$	56 30	181180
$w\,d$ the Transverse Axe		567613	$w\,g$ Focal Distance		247186
* Semi-transverse Axe		283806	p = Semi-focal Distance		123593

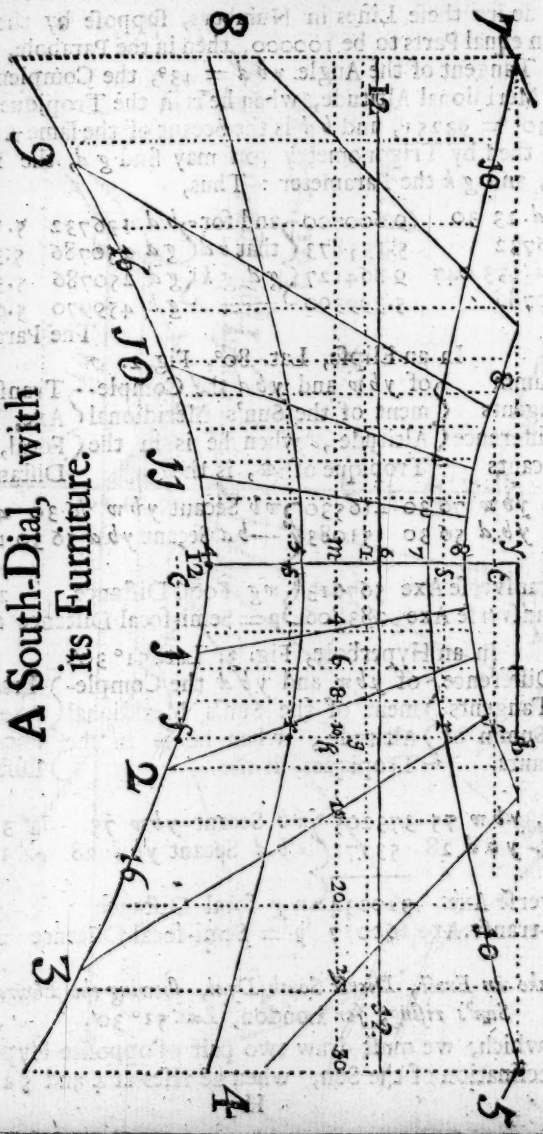
In an Hyperbole, Fig. 3. Lat. 51° 30′.

$w\,d$ the Difference of the Tangents. $w\,g$ the Summ of the Secants. — of $y\,b\,w$ and $y\,b\,d$ the Complement of the Sun's Meridional Altitude, when he is in the Tropiques, is the — Transverse Axe, Focal Distance.

$w\,y$ Tang. $y\,b\,w$	75	373205	$w\,b$ Secant $y\,b\,w$	75	386370
$y\,d$ Tang. $y\,b\,d$	28	53171	* $b\,d$ Secant $y\,b\,d$	28	113257
$d\,w$ Tranverse Axe		320034	$w\,g$ focal Distance		499627
* = Semi-transv. Axe		160017	p = Semi-focal Distance		249813

How to make an Erect, Direct South Dyal, shewing the Hours of the Sun's rising, for London, *Lat.* 51° 30′.

To do which, we must draw two pair of opposite Hyperboles for the Declination of the Sun, when he rises at 4 and 5 a Clock.



A South-Dial, with its Furniture.

First, By Trigonometry find what Declination the Sun has, when he riſes at 4 and 5 aclock, Latitude 51° 30, thus,

Radius	
Co-tangent 51 30	9.900605
Sine 30 for 4	9.698970
Tangent	9.599575
Suns Declin. 21 41 for 4 and 8.	
Sine 15, for 5	9.412996
Tang. of Suns Declin.	9.313601
11° 38′ for 5 and 7.	
38° 30′ Co-elevat. of the Pole,	
21 41 Declination of the Sun.	

Having found the Suns Declination when he riſes at 4, to be 21° 41′, at 5 to be 11° 38, then we are by the foregoing Method to find the Meridional Altitude, by adding and Subſtracting 21 41, to and from the Co-elevation of the Pole, then you have the Tranſverſe Axe and Focal Diſtance.

Summ 60 11 } *y b w* } The Suns Meridianal Altitude when he riſes, as 4 aclock.
Differen. 16 49 } *y b d* } 8 aclock.

Tang. *y b w* 60 11	174492	Secant *y b w* 60 11	201115
Tang. *y b d* 16 49	30223	Secant *y b d* 16 49	164467
Tranſverſe Axe	144269	Focal Diſtance	305582
* = Semi-tranſv. Axe	72134	*p* = Semi-focal Diſtance	152791

Having found the Tranſverſe Axe, and the Focal Diſtance, we muſt ſeek the conjugate Axe, by taking the difference of the Squares of the Focal Diſtance and tranſverſe Axe, which is the Square of the the Conjugate Axe: Or by Logarithms take half the Summ of the Logarithms of the Summ and Difference of the Focal Diſtance and tranſverſe Axe, that is the Logarithm of the conjugate Axe.

Let * = Semi-tranſverſe Axe: *p* = Semi-focal Diſtance *s* = Semi-conjugate Axe.

		Logarithm
p + *	224926	5.352039
p − *	80656	4.906625
pp − ** = SS =	181411	10.258664
or S =	134690	5.129332

After the same manner we must find the Transverse Axe, Focal Distance and Conjugate Axe for 5 and 7, thus,

38° 30′ Co-elevation of the Pole
11 38 Declination of the Sun.

Summ	50 8	$= ybw$	the Sun's Meridional Altitude, when he rises at	5 a clock
Differ.	26 52	$= ybd$		7 a clock

Tang. ybw	50 8	119740	Secant ybw 50 8		156005
Tang. ybd	26 52	50660	Secant ybd 26 52		112100
Transverse Axe		69080	Focal Distance		268105
x = Semi-transverse Axe		34540	p = Semifocal Dist.		134052

$p + x$	168592	5.226838
$p - x$	99512	4.947878
$pp - xx = SS$	$= 167771$	10.224716
$\sqrt{\ }$	$S = 129527$	5.112358

Having found the principal Lines of each pair of Hyperboles, we are next to find the Points of the Curves: In the Rectangle Parallelogram $mhcf$. m is the Centre of the Curves; $cf = mh$, an Ordinate, let g be its Symbol, $mc = hf$ the Distance of the Ordinate from the Centre or Conjugate Axe; let n denote it, having either of these with the Axes, the other is found by the Conicks. The most usual way is, having the Axes and Central Distance, to find the Ordinate; but for the easier delineating these Curves, is, having the Axes and the Ordinate to find the Centural Distance, for less Lines are required for delineation; the same Line hf serving for the other pair of Hyperboles, the Rule is.

Sine $x :: \sqrt{(ss + gg)}\ n$, that is to say, as the conjugate Axe is to the Transverse, so is the $\sqrt{}$ of the Square of the Semiconjugate Axe, and the Square of the Ordinate to the Central Distance: By which Analogy, I have calculated the Central Distance $n = hf$ for several Ordinates, as they are here set down, both for 4 and 8 a clock, as also for 5 and 7 a clock, and for the Tropiques; of each I will give you two Examples.

First

First say, as $s.\ *\ ::\ it$, the ratio

$* =$	7, 2134	0.858146
$s =$	13. 469	1.129332
$t. = \overset{*}{s}. =$	0.5356	1.728814 for 4 and 8.

Let $g =$ 4
To ss 181 411
Adde gg 16

$ss + gg$	197 411 Whose Logarithm is	2.295371
Whose Root extract by dividing by 2		1.147658
Add the common Logarithm of t, the ratio		1.728814
You have the Logarithm of $n = 7{,}525$.		0.876498

Let $g = 20$
To ss 181 411
Adde gg 400

$ss + gg$	581 411 whose Logarithm is	2.764488
Whose Square Root, extracted by dividing by 2		1.382244
Add the common Logarithm of t, the ratio		1.728814
You have the Logarithm of $hf = n = 12\ 91$		1.111058

For 5 and 7, say, as $s\ *\ ::\ it$ the ratio.		Logarithm
$* =$	3.454	0.538322
$s =$	12 9527	1.112358
$t = \overset{*}{s} =$	0 2666.	1.425964.

Let $g = 4$.
To ss 167 771
Add gg 16

$ss + gg$	183 771 whose Logarithm is	2.264274
Extract its Square Root by dividing by 2		1.132137
Add the common Logarithm of the Ratio		1.425964
You have the Logarithm of $gr = n = 3\ 615$		0.558101

Let

Let $g = 15$		
To ss	167 771	
Add gg	225	
$ss + gg$	392 771 whose Logarithm is	2.594148
Whose Square Root extracted by dividing by 2		1.297074
Add the common Logarithm of t, the Ratio		1.425964
You have the Logarithm of $gr = n = 5285$		0.723038

The Central Distance for several Ordinates, for both pair of Hyperboles for 4 and 8 a clock, for 5 and 7 a clock, and for the two Tropiques.

		for 4 and 8 a clock	for 5 and 7 a clock	for the two Tropiques.
	*	7 2134 *	3 454 *	8 0639
g	1 n	7 233 n	3 465 n	8 085
g	2 n	7 292 n	3 494 n	8 15
g	4 n	7 525 n	3 615 n	8 404
g	6 n	7 897 n	3 806 n	8 812
g	8 n	8 390 n	4 059 n	9 538
g	10 n =	8 981 n =	4 364 n =	10 004
g =	12 n	9 655 n	4 708 n	10 74
g	15 n	10 797 n	5 285 n	12
g	20 n	12 91 n	6 354 n	14 32
g	25 n	15 209 n	7 504 n	16 85
g	30 n	17 612 n	8 714 n	19 51
g	36 n	20 585 n	10 202 n	22 79
g	40 n	22 60 n	11 21 n	25 03

How to delineate these Curves.

Let by be perpendicular to the Meridian; $yb = 10$ Inches, the length of the Pin that shews the Hours.

Make $y8$ = Tangent 16° 49′ = 3 02 Inches for the Vertex of the Hyperbole when the Sun rises at 8 a clock, and $y4$ = Tangent 60° 11′ = 17 45, for the Vertex of the opposite Hyperbole, when the Sun rises at 4 a clock; their difference is the transverse Axe = 14 426 Inches, which if you bi-sect in m, you have their Centre, where erect a Perpendicular to the Axe, which is the Con-

Conjugate to measure the Central Distance, as $m h$; and at 4 6 8 10 12 15 20 25 30 36 40 Inches from the Centre draw parallel Lines to the Axe, on each Side thereof, then take the Numbers of the Central Distance belonging to each Ordinate, as you find them in the Table, in the foregoing Page; as for $g = 4$, take $n = 7\ 52$ Inches, and place them from the conjugate Axe $m h$, upward and downward for the Points of the Hyperboles, the Ordinate being 4 Inches; so if the Ordinate $g = w$ take $n = 8\ 98$ Inches, and place it both ways from h, as $h f$ for other Points, and so proceed till you have gone over all the Ordinates for 4 and 8 a clock; then with a steady Hand joyn these Points, and you have the Curves desired.

To describe the Hyperboles for the Sun's rising at 5 and 7 a clock, make on the Axe $y 7$ = Tangent $26° 52' = 5\ 06$ Inches, the Sun's Meridional Altitude, when he rises at 7 a Clock, and 75 = Tangent 50 *degrees* 8 = 11.97 Inchesthe Suns Meridional Altitude, when he rises at 5 a clock; then at 7 and 5 are the Vertexes of the said Curves, and their Distance 69 08 Inches is their transverse Axe, which if you bi-sect in i, you have their Centre, where erect a Perpendicular to the Axe, as $i g$, which is the conjugate Axe, and Line to measure the central Distances; then on the Parallels to the Axe already drawn, measure the central Distances noted in the Table by n, for each Ordinate, as if $g = 4$, then take $n = 3\ 61$ Inches, which place on both Sides of the Axe upward and downward; so if the Ordinate $g = 10$, then $n = 4\ 36$ Inches, on both Sides of the Axe both ways, as $g r$, and so on each Ordinate g, place n, it's Central Distance, and through those Points draw the Curves desired.

To draw the Equator, make on the Axe $y 6$ = Tangent $38° 30' = 7\ 95$ Inches, where draw a Perpendicular to the Axe, that is the Equator.

Forasmuch as the Centre is not on the Plane, the Tangent of $51° 30' = 12\ 57$ Inches gives you the Centre upward, and the Secant of $51° 30' = 16\ 06$ Inches, the Distance of the Tangent from the Centre, and the difference between the Secant and Tangent of $51° 30' = 3\ 49$ Inches, directs us to draw the first Tangent so many Inches below y, and 16 06 Inches below that the Second Tangent, and then mark the Hours by the Tangent, as on a Horizontal Dyal; joyn those Marks and you have the Hours.

If

If you deſire to draw the Tropiques, make y ♈ = Tangent 15 = 268 Inches, the Sun's Meridianal Altitude when he is in the Tropiques of ♈, and y ♋ = Tangent 62 = 1880 Inches, the Suns Meridional Altitude when he is in the Ttropique of ♋, then ♈ and ♋ are Vertexes of the pair of Hyperboles, and their Diſtance 1612 Inches is the tranſverſe Axe, which if you bi-ſect, you have the Centre, through which apply a Perpendicular to the Axe, which is the conjugate Axe to meaſure the central Diſtance of each Ordinate, as you will find by the Table, page 15. By the foregoing Method, you will find x the Semi-tranſverſe Axe to be 806 Inches, p the ſame ſemi-focal Diſtance to be 1582 Inches, s the ſemi-conjugate Axe to be 1362, and ss it's Square, to be 185454.

CHAP. VI.

How to find the Inclination of the Plane.

TAke a Quadrant, and joyn it to the Side of a Square, and place the other Side of your Square to your Plane; if the Plummet fall parallel to the Side of the Square, then the lower Side of the Square ſtands level, by which draw an Horizontal Line, on which erect a Perpendicular, and apply your Square to that Perpendicular, and if the Plummet falls parallel to the Side of the Square, then that is alſo a level Line, and your Plane ſtands Horizontally: If the Plummet falls not parallel to the Side of the Square, then turn your Square until it does, then draw an horizontal Line, on which erect a Perpendicular, to which apply your Square, and obſerve what Angle your Plummet makes on the Quadrant, with the Side of the Square, that is the Angle of the Inclination of the Plane.

How to find the Declination of a Wall by an Analemma.

To deſcribe an Analemma for Lat. 51° 30′

If you have a Line of Chords and Sines for one Radius, you may uſe that inſtead of the Table; the longer the Radius the better.

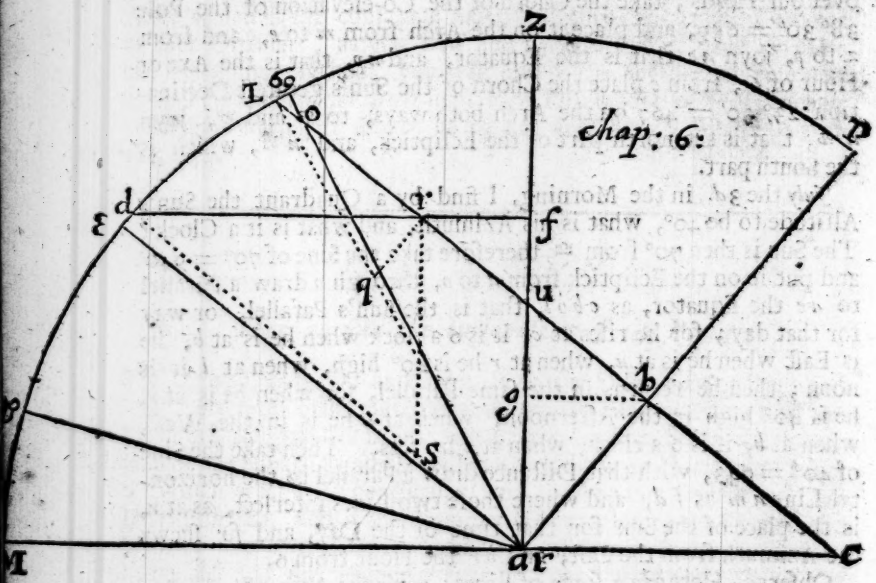
Deſcribe

Deſcribe the Arch of a Circle *p z l m* for the Meridian, whoſe Radius or Semi-diameter is equal to 10 Inches, or 10 half Inches; through the Centre draw the Diameter *a m* for the Horizon, *a* is the Centre of the Arch, at *a* erect the Perpendicular *a z*, which is the prime Vertical, or Line of Eaſt and Weſt, and at *z*, where it cuts the Meridian, is the Zenith, being the Point over our Heads; take the Chord of the Co-elevation of the Pole 38° 30′ = 659, and place it on the Arch from *m* to *e*, and from *z* to *p*, joyn *a e* that is the Equator, and *a p*, that is the Axe or Hour of 6; from *e* place the Chord of the Sun's greateſt Declination 23° 30′ = 467 on the Arch both ways, to ♋ and ♑, joyn *a* ♋, that is the north part of the Ecliptick, and *a* ♑, which is the South part.

July the 3*d.* in the Morning, I find by a Quadrant the Sun's Altitude to be 40°, what is his Azimuth, and what is it a Clock? The Sun is then 70° from ♎, therefore take the Sine of 70° = 940′ and put it on the Ecliptick from *a* to *o*, through *o* draw a Parallel to *a e* the Equator, as *c b o l*, that is the Sun's Parallel, or way for that day; for he riſes at *c*, it is 6 a clock when he is at *b*, he is Eaſt when he is at *u*, when at *r* he is 40° high, when at *l* it is noon; then he returns in the ſame Parallel, for when he is at *r*, he is 40° high in the Afternoon, when at *u* he is in the Weſt, when at *b*, it is 6 a clock, when at *c* he ſets. Then take the Sine of 40° = 643, with that Diſtance draw a Parallel to the horizontal Line *a m* as *f d*, and where thoſe two Lines interſect, as at *r*, is the place of the Sun for that time of the Day, and *f r* ſhews the Azimuth from the Eaſt, and *b r* the Hour from 6.

Obſerve, Here are 3 ſorts of Lines: Firſt the Arch of a Circle; Secondly, Streight Lines paſſing through the Centre; Thirdly, ſtreight Lines, which do not go through the Centre. The Arch of the Circle is to be meaſured with a Line of Chords. Secondly, The ſtreight Lines which paſs through the Centre, which on the Globe are great Circles bi-ſecting the Globe, are to be meaſured by a Line of Sines, according to the Radius of the Circle, as is the Sun's Declination, and its Altitude; therefore if at any time of the Day, you deſire to know its Altitude, take its neareſt Diſtance to the Horizontal Line *a m*, and meaſure it on the Line of Sines, and you have your deſire. Thirdly, The ſtreight Lines which go not through the Centre, and are the leſſer Lines on the

Globe, having no Line of Sines for those short Diameters, you must draw a Semi-diameter to their Intersection with the Meridian, as the prickt Lines *a l*, *a d*. If you seek the Sun's Azimuth, make *r s* parallel to *f a*; then *a s* = 278, the Sine of 15 40, the Sun's Azimuth from the East, for *d f*, *d a* :: *r d*, *s d* :: *f r*, *a s*, being like Triangles.

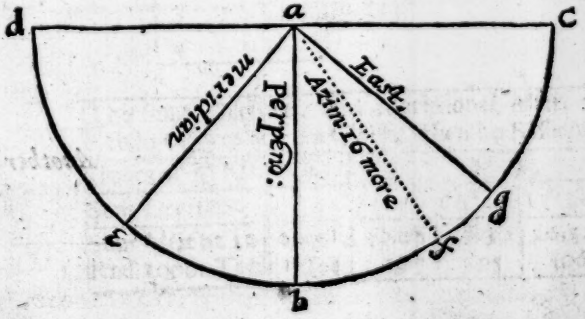


If you seek the Hour, make *r q* Parallel to *a b*, then *a q* = 606 is the Sine of 37°, which converted into Time, accounting 15° to an Hour, is 2 Hours 28 Minutes from 6 = 8 Hours, 28 Minutes for *b l*, *a l* :: *b r*, *a q* :: *r l*, *l q*. If you had rather work by Numbers, say,

As *f d* the Co-sine of 40° = 7660 is to *a d* 10000,
so is *f r* 208 to *a s* 271 the Sine of 16°, the Sun's Azimuth.
And as *b l* 9272, the Co-sine of the Sun's Declination is to *a l* 10000,
so is *b r* 564 to *a q* 608, the Sine of 37°.

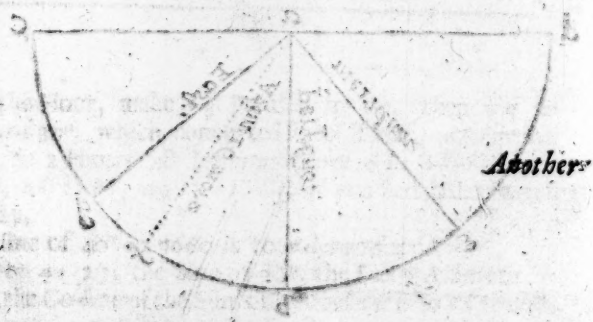
The

The Arch of the Circle is the Meridian.
c a m The Horizon, *z* The Zenith.
z a The Prime Vertical, or Line of East and West.
a p The Axe of the World, or Hour of 6.
z p The Co-elevation of the Pole = 38° 30'.
c a p = Elevation of the Pole = 51° 30'.
e a The Equator, *a* Aries or Libra.
a Cancer, *a* Capricorn } the { North, South } part of the Ecliptick.
e ♈ and *e* ♎ = 23° 30', the Sun's greateſt Declin.
o the Sun's place in the Ecliptick = 70° from ♈.
a b the Sine of the Sun's Declination.
l e The Arch of the Sun's Declinaſion = 22°.
b o The Sine of the Sun's Right Aſcention
a c The Sine of the Sun's amplitude.
b c The Sine of the Sun's aſcenſional Difference.
m l The Sun's Altitude at Noon.
a u The Sine of the Sun's Altitude in the Eaſt.
b u The Sine of the Time from 6, in the Eaſt.
a g The Sine of the Sun's Altitude at 6 a clock.
b g The Sine of the Sun's Azimuth at 6 a clock.
f d A Parallel of the Sun's Altitude, being 40° high
r The Place of the Sun for that time.
f r The Sine of the Sun's Azimuth from the Eaſt.
b r The Sine of the Time from 6.



At the ſame Time that you take the Suns Altitude by the Quadrant, put a Quadrant or Semicircle to the Wall Horizontally; take a Line and Plummet, which move by the Edge of the Quadrant, till the ſhade of the Line paſſes through the Centre of the Quadrant, then the Degree in the Limb ſhews what Angle the Sun at that inſtant makes with the Wall: If you have no Inſtrument, apply a ſtreight Board to the Wall, and mark at two ſeveral places the ſhade of the Line, then joyn thoſe Places, and meaſure the Angle by a Line of Chords, that it makes with the Side of the Board, which is the Angle the Sun makes with the Wall.

Then on Paper deſcribe a Semicircle *a b c d*, which croſs at Right Angles with *a b*, which ſuppoſe to be the Inſtrument applyed to the Wall *July* 3, the Azimuth being found to be 16° paſt the Eaſt, and at the ſame time the prickt Line *a f* to be the Shade of the Sun, then with a Line of Chords, on the Circle place *f g* = *i b* backward towards *c*, for that he is paſt the Eaſt; joyn *a g*, that is the Eaſt, and the Angle *g a c* is the Declination of the Plane; or put *f e* the Complement of the Sun's Azimuth = 74 forward towards *d*; joyn *a e*, that is the Meridian, and the Angle *b a e* is equal to the Angle *g a e*, the Declination required.



Another

Another easie Method from Tables; to describe the Curves on each Hour; shewing the time of Suns Rising, and Length of the Day, on any Upright Dyal: The following Tables are made for Lat. 51. deg. 32 min. *but by the same Rules may be accomodated to any Latitude.*

A Table of the Suns Azimuth for each Hour; Latitude 51 *deg.* 30. *min.* when he Rises at 4, 5, 6, 7 and 8 a Clock; with their Tangents, the Rad. 1000.

Hours of ⊙ Rising	4	5	6	7	8
Hours 12	o° o′	o° o′	o° o′	o° o′	o° o′
11 1	26.58	22.23	18.53	16.18	14.27
	509	412	342	292	251
10 2	49.17	42.17	36.25	31.49	28.23
	1.162	909	737	620	540
9 3	66.36	58.59	51.57	46.03	41.27
	2.310	1.663	1.277	1.037	883
8 4	80.35	73.06	65.41	59.03	53.35
	6.029	3.291	2.213	1.667	1.355
7 5	92.42	85.36	70.09	71.06	
		12.996	4 760	2.920	
6 6	103.54	97.18	90		
5 7	114.57	10.854			
4 8	126.25				

The Suns Declinat. and Meridional Altit. for Latitude 51° 30′ at 12 a Clock, when he Rises at ⟶

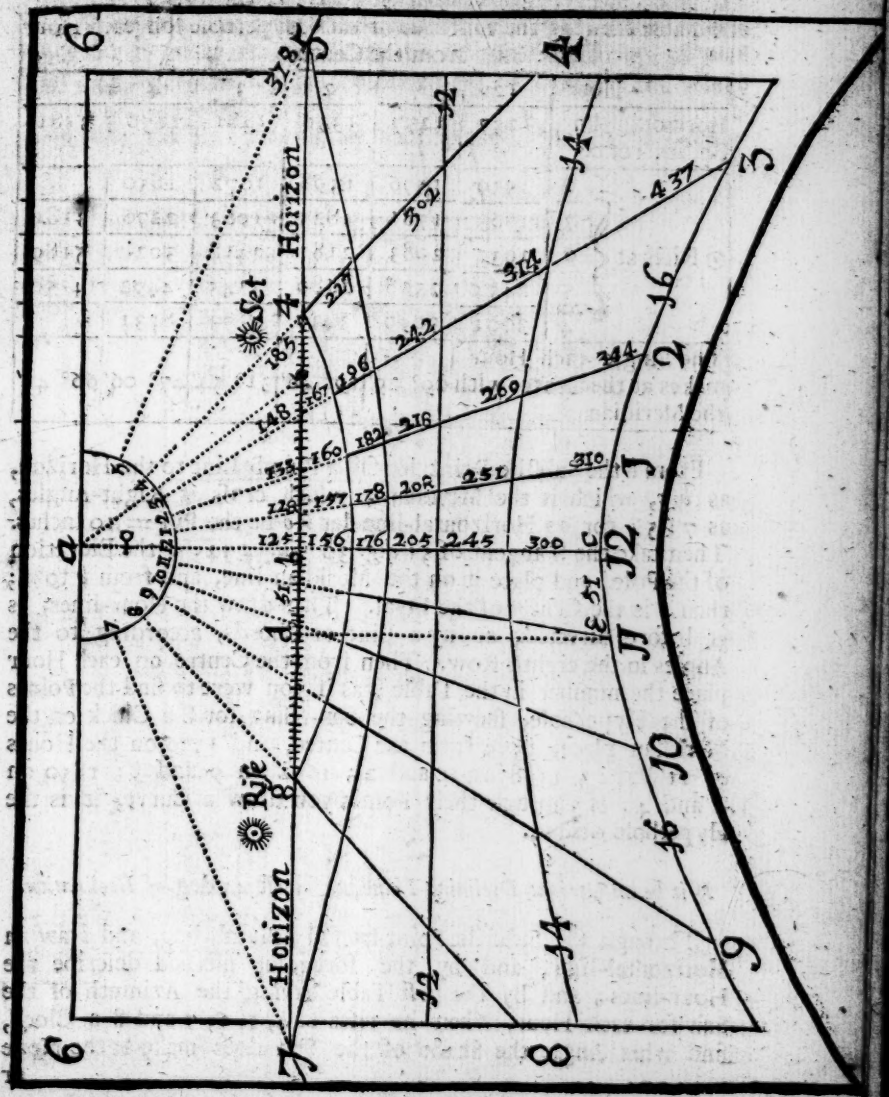
Hours of ⊙ Ris.	4	5	6	7	8
Suns Declinat.	21° 41′	11° 38′	00° 00′	11° 38′	21° 41′
Suns Altit. at 12	60.11	50.08	38.30	26.52	16.49
Rad. 1000. Tan.	1.744	1.197	795	506	302

If it be an Erect Direct South-Dyal, and the Pin giving the Shadow ten inches in length for the hour of 12, you must take the Tangent of the Suns Meridional Altitude, as in the second Table, and place it on the Meridional-line downward from the Substile Point, as the Tangent of *60 deg.* 11 *min.* = 17,44 when the Sun rises at Four of the Clock. And so for the other Hours of Sun rising.

Then for the other Hours of the Day, you must conceive all the Azimuths on upright Planes, to be parallel to the Meridian; and where the Hour and the Azimuth of that Hour intersects, to be one Point of the Curve desired; as for 11 and 1 a Clock, the Sun rising at 4, the Suns Azimuth is *26 deg.* 58 *min.* whose Tang. 509 being placed on the Horizontal-line from the Meridian, as *b d*, and then a Parallel-line drawn to the Meridian, as *d e*; at *e* where it cuts the Hours of 11, is one Point of the Hyperbole. Then thorough their Points thus found a Curve being drawn, it is the Hyperbole desired.

Or if you had rather, you may find the distance of each Curve on each Hour from the Centre by the ensuing Table, being made from the former Tables, by the Doctrin of plain Triangles; for in the Triangle *a c e* there is given the Angle at the Centre, it being the Angle that each Hour makes at the Centre of the Dyal with the Meridian on an Erect Direct South Plane found by Calculation, it being the eighth Row in this Table: One leg of the Triangle *e c* being the natural Tangent for the Suns Azimuth for that Hour, as in the first Table, and the Hypothenuse is sought after; which is the distance of the Hyparbole on that Hour from the Centre.

If there be no Centre on the Plane, you may measure the distance on this Hour from the Horizontal-line (for which end it was here inserted) by taking the difference between the numbers of the Hour and the Horizontal-line, as for the Curve of the Suns rising at 8 a Clock for the Hour of 12. 1559 —1257=302, the distance on the Meridian between the Horizontal-line and the Curve; so 1579 — 1277 =299 for 1 a Clock; and 1598—1336 = 262 for 2; and 1672 — 1481 = 191 for 3. And so for the other Hours of Sun-rising.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A Table shewing the Distance of each Hyperbole on each Hour from the Centre.

Hour:	12	1	2	3	4	
Horizontal-line, Radius 1000.	1257	1277	1336	1481	1850	3181
⊙ Rises at 8	1559	1576	1598	1672	1850	
⊙ Rises at 7	1763	1781	1825	1963	2276	3181
⊙ Rises at 6	2052	2083	2181	2418	302,	5189
⊙ Rises at 5	2454	2508	2689	3145	4492	14150
⊙ Rises at 4	3001	3099	3437	4373	8231	
The Angle each Hour makes at the Centre with the Meridian	09° 29′	19° 46′	31° 54′	47° 06′	66° 42′	

From *b* the Substile Point let fall a Plumb Line to the Horizon, as *b c*, which is the Meridian; which cross at Right-Angles, as 7 *b* 5; for an Horizontal-line let *b o* be the Pin = 10 Inches. Then take the Tangent of 51 *deg.* 30 *min.* = 1257, the Elevation of the Pole, and place it on the Meridian-line, and from *b* to *a*; then *a* is the Centre of the Dyal. Then draw the Hour-lines, as is before directed, or by a Line of Chords, according to the Angles in the eighth Row. Then from the Centre on each Hour place the number in the Table; as if you were to find the Points of the Hyperbole, shewing the Sun-rising for 8 a Clock on the Meridian place, 1559 from the Centre, and 1576 on the Hours of 11 and 1; 1598 on 10 and 2; 1672 on 9 and 3; 1850 on 8 and 4: If through their Points you draw a Curve, it is the Hyperbole desired.

If it be an Upright Declining Plane, as suppose 45 deg. *of Declination.*

Through the Substile Point let fall a Plumb-line, and draw an Horizontal-line, and by the foregoing method describe the Hour-lines; and by the first Table having the Azimuth of the Sun for each Hour, when he rises at 4, 5, 6, 7 and 8 a Clock, find what Angle the Shade of the Sun does make at the Node

or

or Vertix of the Pin with the Pin, by adding and ſubſtracting the Suns Azimuth to and from the Planes Declination; as for 11 and 1, when the Sun riſes at 4 a Clock, its Azimuth is 26 *deg.* 58 *min.* Then 45 *deg.* + 26 *deg.* 58 *min.* = 71 *deg.* 58 *min.* for 1, whoſe Tangents are 325. And by this method the enſuing Table is made. 30,7. 18 2 11

A Table of the Angles that the Shade of the Sun does make each Hour at the Vertex of the Pin, with the Pin when the Sun Riſes at 4, 5, 6, 7 and 8 a Clock, with their Tangent, except 12, the Radius being 1000.

Hours	4	5	6	7	8
10 2	00.00	87.17	81.35	76 49	73.23
		21.074	6.758	4.270	3.350
11 1	71.58	67 23	63.53	61.18	29.27
	3.070	2.400	2.040	1.826	1.694
1 11	18.02	21.37	26.07	78.42	30.33
	325	416	490	547	590
2 10	04.19	02.43	08.35	13.11	16.37
	75	47	151	234	298
3 9	21.30	13 59	06.57	01.03	03.33
	396	249	122	18	62
4 8	35.35	28.06	20.41	14.03	
	715	534	1.377	250	
5 7	47.40	40.46	33.09		
	1.097	862	653		
6 6	58.54	52.18			
	1.657	1.294			
7 5	69.59				
	2.745				

Then by placing their Tangents on the Horizontal-line from the Subſtile, and drawing Parallel-lines to the Meridian at their

Interſection, with their reſpective Hours, are found the Points of the deſired Hyperbole.

And to find the diſtance of the Sun on the Meridian from the Horizontal-line; ſay, as the Radius is (Fig. VIII.) to *m e*, the Secant of the Angle *d e m* the Planes Declination, ſo is the Tangent of the Suns Meridional Altitude, when he riſes at 4, 5, 6, 7, and 8 a Clock. To find the diſtance of the Curves on the Meridian from the Horizontal-line.

Radius			10.000000
Secant, Suns Declination 45 *deg.*	14142.		4.150525
Tangent, Suns Meridional Altitude 60 *deg.*	11.		10.241776
Suns Diſtance on the Meridian	24680		4.342301

After the ſame Analogy, you will find the diſtance of the Hyperbole on the Meridian to be 1693 for 5, and 1125 for 6, and 716 for 7, and 427 for 8 a Clock.

Soli Deo Gloria.

THE